ASMo
Manual

(v0.3.0)

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1 Introduction

ASMo is a collaborative research project aimed at developing a user-friendly software environment for the modeling and analysis of stochastic input models. The software is built on open-source components and incorporates methods contributed by the team members, with a primary focus on flexibility and performance. ASMo facilitates the creation of stochastic models from data, which can be seamlessly integrated into larger simulations or utilized independently. The project involves the development of novel methods, refinement of existing approaches, and practical application in real-world scenarios.
This document is structured as follows. Installation instructions for ASMo are given in Sect. 2. Sect. 3 contains an introductory tutorial and Sect. 4 explains the use of ASMo in detail.

2 Installation

This section describes the installation of ASMo and the steps you need to follow.

2.1 System Requirements

1. **Operating System:** This guide assumes you're using Windows, macOS, or Linux.

2. **Basic Command Line Knowledge:** You'll need some familiarity with your operating system's command line (e.g., Terminal on macOS, Command Prompt or Powershell on Windows).

3. **Install Python:**
   Visit the official Python download site `https://www.python.org/downloads/` and download the appropriate installer for your operating system. Run the installer and follow the on-screen instructions.

4. **Install pip:**
   `pip` usually comes bundled with Python. To verify, open your command line and type `pip --version`. If it's not installed, follow the instructions found on `https://pip.pypa.io/en/stable/installation/`.

5. **Install Poetry:**
   Open your command line and type `pip install poetry`.

2.2 Steps of Installation

There are three possible methods for installing ASMo: via Git clone (if you have access to the repository), using the zip file, or downloading the software from the ASMo webpage. Follow one of the three methods based on your situation.

2.2.1 Method 1: Git Clone (Repository Access)

If you have access to the ASMo repository, you can use Git to clone the project and set it up.

1. **Clone the project:**
   Ensure you have Git installed by checking with `git --version`. Navigate to the directory where you want to store the project, then clone the repository by using `git clone --recurse-submodules [ASMo project URL]`.

2. **Switch to the cloned directory:**
   Navigate to the project folder using the `cd` command. For example, `cd ASMo` (replace `ASMo` with the actual folder name).

3. **Install dependencies using Poetry:**
   In your command line (while inside the project directory), run `poetry install` to install the project dependencies.

4. **Run ASMo:**
   After installing the dependencies, you can start the application by typing `poetry run asmo` in your command line.

2.2.2 Method 2: Zip File

If you have downloaded the ASMo project as a zip file, follow these steps to install it.

1. **Unzip the ASMo-zip-file:**
   Unzip the downloaded ASMo zip file to your preferred directory.

2. **Install dependencies using Poetry:**
   Navigate to the unzipped project directory in your command line and run `poetry install` to install the necessary dependencies.

3. **Run ASMo:**
   Once the dependencies are installed, start the application by running `poetry run asmo`.

4. **Access ASMo on localhost:**
   Open a local browser and go to `localhost` to access the ASMo interface.

2.2.3 Method 3: ASMo Webpage

If you're installing ASMo by downloading the software from the official ASMo webpage, follow these steps.

1. **Download from ASMo webpage:**
   Go to the official ASMo website and download the software package.

2. **Install dependencies using Poetry:**
   After downloading, open your command line, navigate to the project directory, and run `poetry install` to install the dependencies.

3. **Run ASMo:**
   To start the application, type `poetry run asmo` in your command line.

4. **Access ASMo on localhost:**
   Open a browser and access ASMo by navigating to `localhost`.

3 Quick Start Guide

The following sections explain step-by-step how ASMo's GUI can be used for the modeling and analysis of stochastic input models such as distributions and traces. This quick-start guide will only explain the basic functionalities of a distribution and trace model. A detailed description of ASMo that introduces all the features and properties will be given in Sect. 4.

3.1 Distribution Example

In the following, we will assume that the user wants to add a distribution card and inspect it.
As shown in Fig. 3.1 we will start by adding a new distribution card. At the right corner of the page go to `Card List` and click `ADD NEW CARD`.

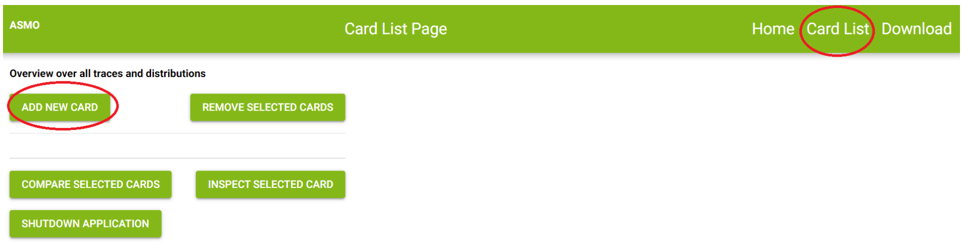


Figure 3.1: Add new card

This will take you to Fig. [3.2, 3.3], where you can `UPLOAD TRACE OR DISTRIBUTION`, or `DEFINE DISTRIBUTION`.

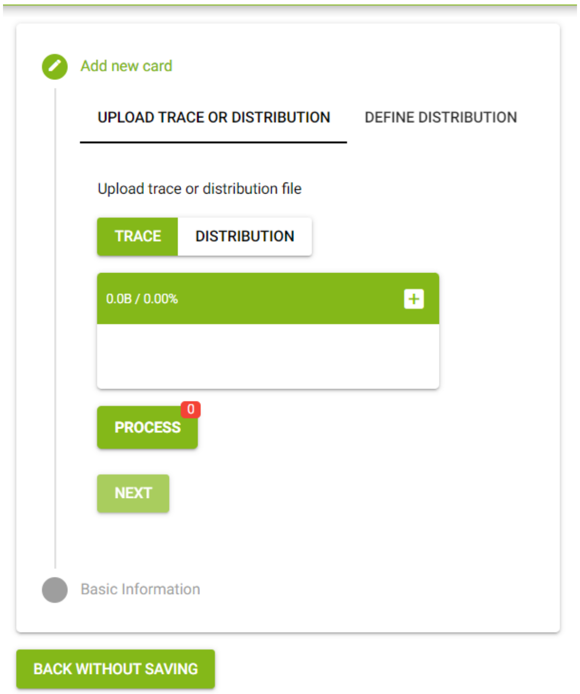


Figure 3.2: Upload trace or distribution

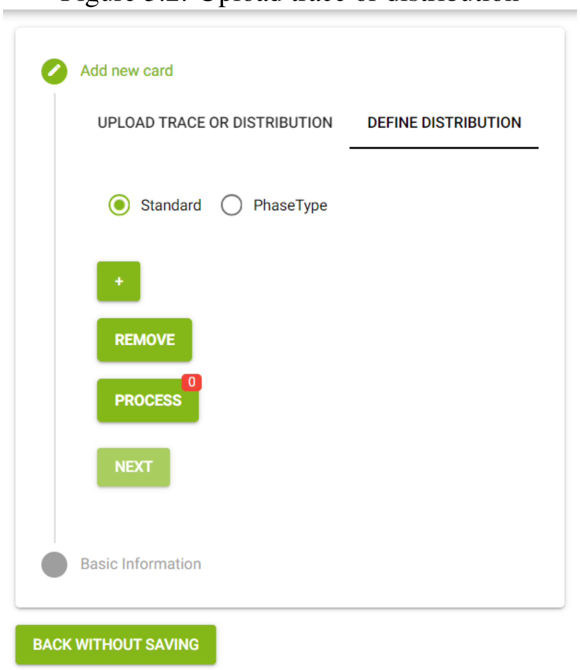


Figure 3.3: Define distribution

Let us try to add a distribution, by using the option `DEFINE DISTRIBUTION`. The defined distribution is a normal distribution with the given parameters, see Fig.3.4.

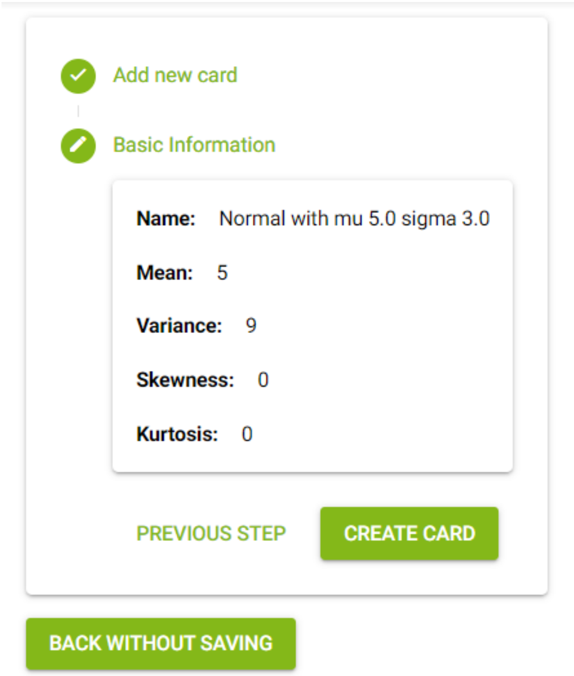


Figure 3.4: Distribution content

Now the created distribution card can be inspected by choosing `INSPECT SELECTED CARD` as shown in Fig. 3.5.

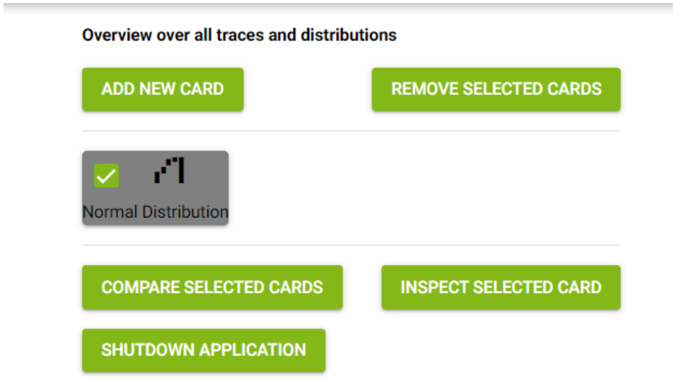


Figure 3.5: Inspect selected card

The options available for inspection are shown in Fig. 3.6.

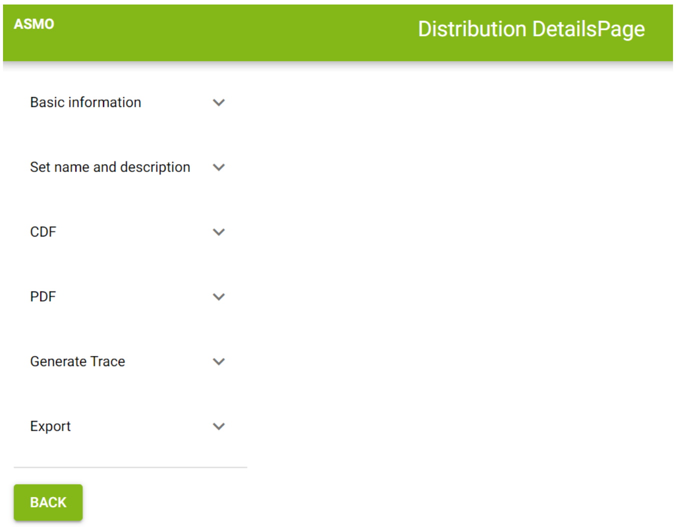


Figure 3.6: Inspect selected card

1. The option `Basic information` as shown in Fig. 3.7 shows all the basic information of the distribution card, such as `Name` , `Type` , `Description` , `Parameters` , `Mean` , `Variance` , `Skewness` and `Kurtosis` .
2. Figure 3.8 illustrates the `CDF` of the distribution card.

Basic information

Field	Value
Name	Normal Distribution
Type	Normal
Description	This is a normal distribution with a mean of 5 and a standard deviation of 3.
Parameters	{'mu': 5, 'sigma': 3}
Mean	5.0
Variance	9.0
Skewness	0.0
Kurtosis	0.0

Figure 3.7: Basic information

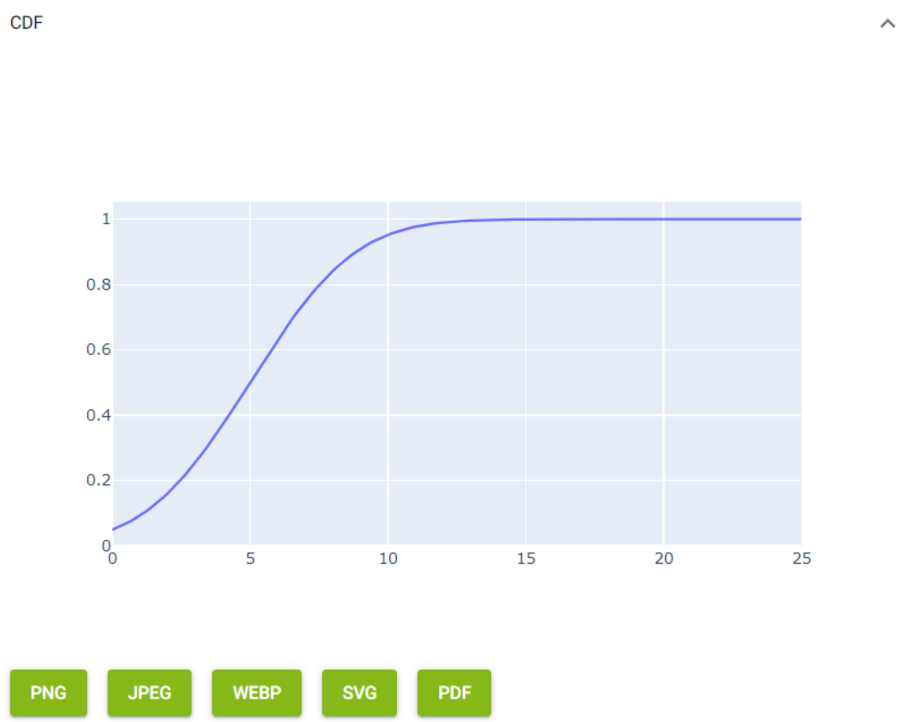


Figure 3.8: CDF of distribution

3. The `PDF` of the distribution card is presented in Fig. 3.9.

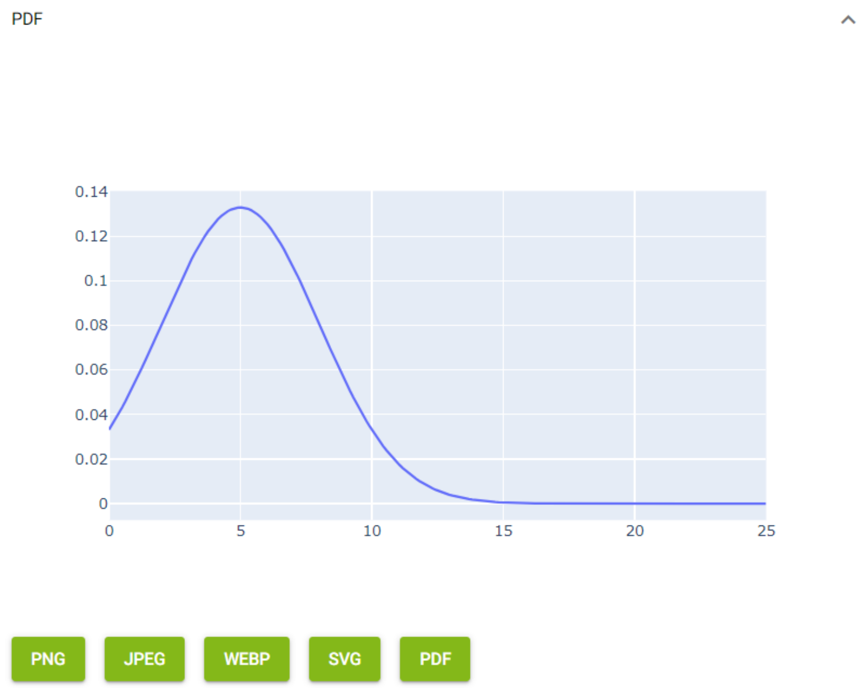


Figure 3.9: PDF of distribution

4. You can `Generate Trace` by specifying the number of elements (in this case, 1000) and then clicking `GENERATE`, as shown in Fig. 3.10.

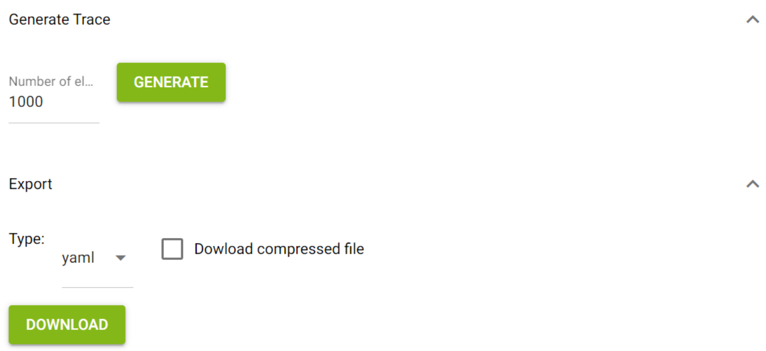


Figure 3.10: Generate trace

3.2 Trace Example

In the distribution example, we generated a trace from the distribution card. Now, let's inspect the generated trace card. The available inspection options are shown in Fig. 3.11.

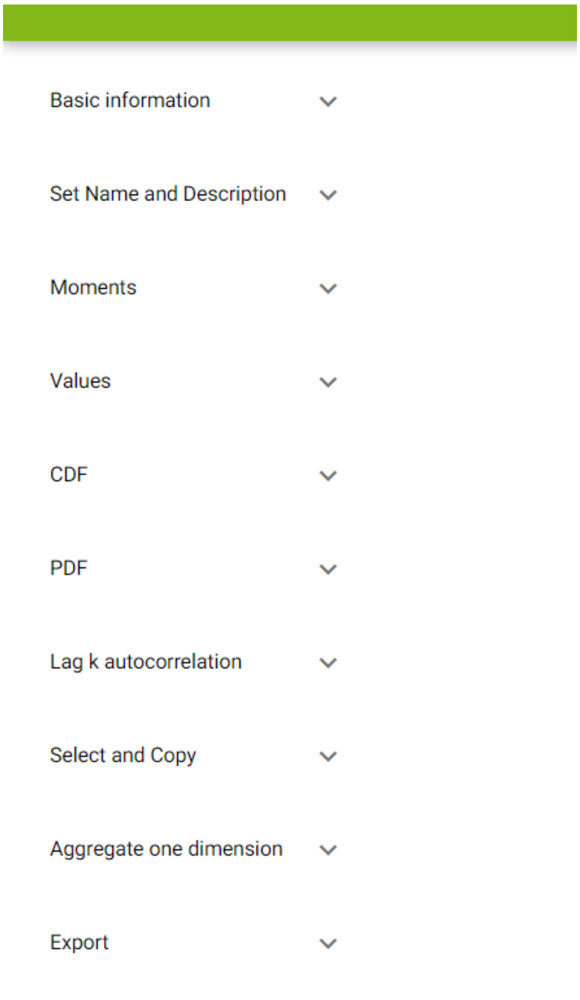


Figure 3.11: Inspect trace

1. As shown in Fig. 3.12, the `Basic information` of the trace includes `Name`, `Type`, `Source`, `Description`, `Dimension` and `No. of Entries`.

Basic information

Field	Value
Name	trc_of_Normal Distribution
Type	Trace
Source	None
Description	None
Dimension	1
No. of Entries	1000

Figure 3.12: Basic information

2. The option `Moments` gives information about the `Dim.`, `Mean`, `Variance`, `Skewness`, `Kurtosis`, `Corr. vert.`, `Minimum` and `Maximum`, see Fig. 3.13.

3. You can also examine the `Values` option, as illustrated in Fig. 3.14, which provides a value plot.

Moments

Dim.	Mean	Variance	Skewness	Kurtosis	Corr. vert.	Minimum	Maximum,
0	4.902668311468829	7.920620008882342	0.046530690147694447	-0.2216247868602399	-0.023937251523933712	-3.737462591421206	13.020544497927025

Figure 3.13: Moments

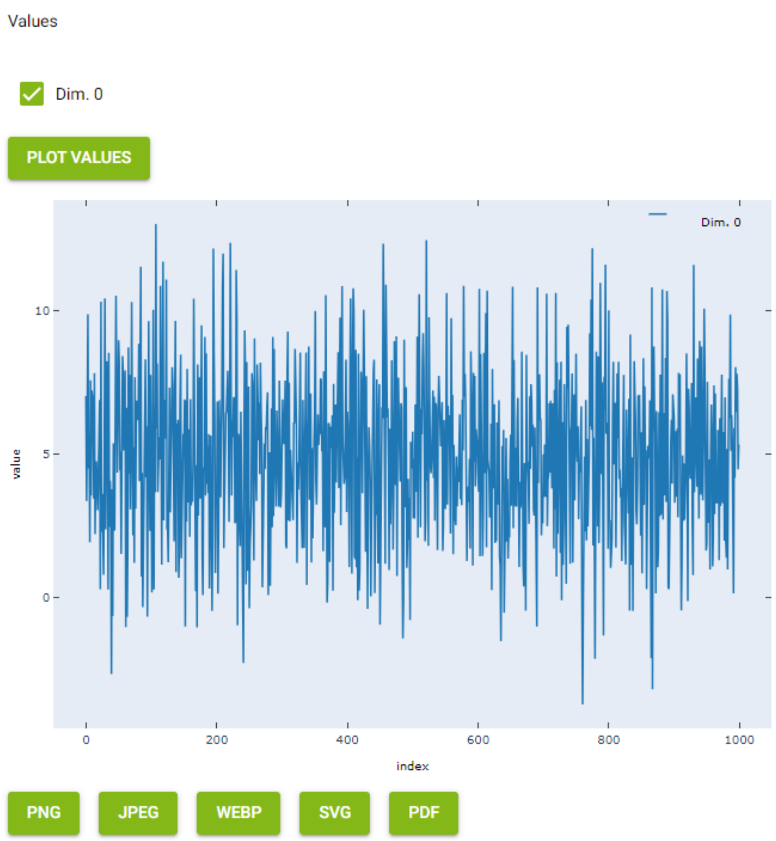


Figure 3.14: Values

4. Figure 3.15 demonstrates how to generate the `CDF`.

Figure 3.15: CDF

5. Figure 3.16 demonstrates how to generate the `PDF`, allowing you to specify the `number of bins`.

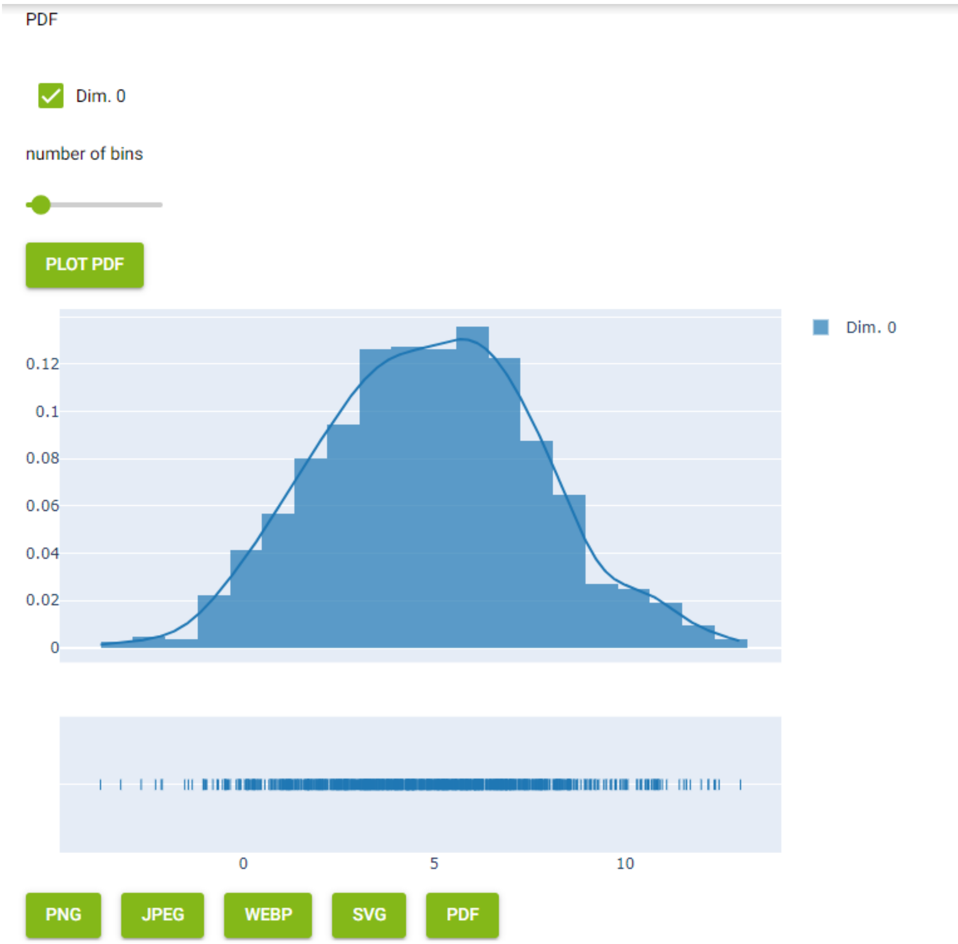


Figure 3.16: PDF

6. You can also examine `lag k autocorrelation` with `lag k autocorrelation` set to 50, as shown in Fig. 3.17.

Figure 3.17: Lag k autocorrelation

7. As illustrated in Fig. 3.18, you can `Select dimension to aggregate` and choose the number of `bins`. Let's set the number of `bins` to 10. When you click `AGGREGATE`, a new card will be created.

8. At the end you can choose to `FIT DISTRIBUTION OR PROCESS` as demonstrated in Fig. 3.19.

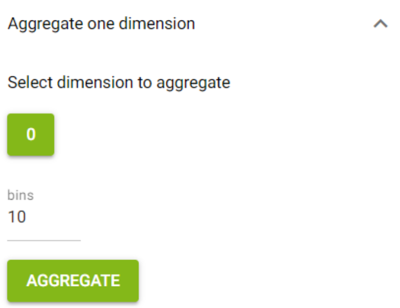


Figure 3.18: Aggregate one dimension

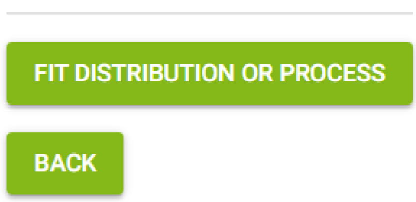


Figure 3.19: Fit distribution or process

3.3 Fit a Distribution

After choosing `FIT DISTRIBUTION OR PROCESS`, you will be redirected to the following page: `Fitting Procedure – One Dimension`, see Fig. 3.20. At the `Trace` tab you can select `Trace` and `Dimension`.
At the `Fitting` tab, you can select `Select model`, where you can select `Kind` (in our case it is a distribution) and `Type`.
Then you can choose `START FITTING`.

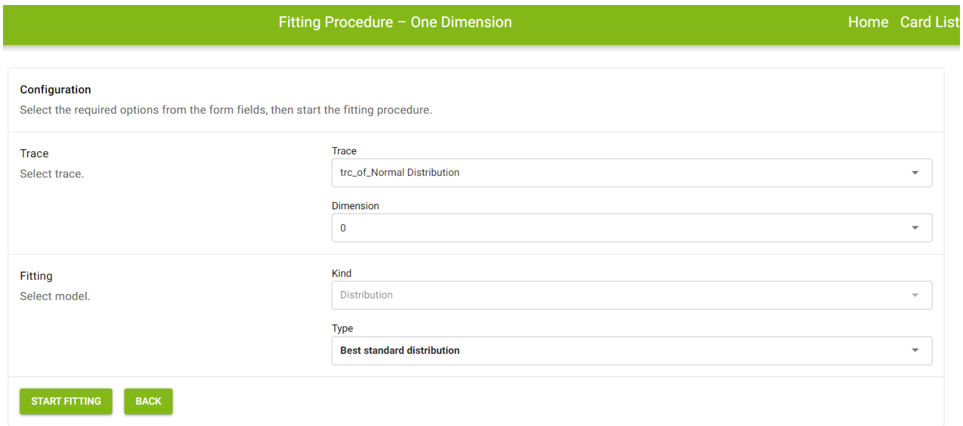


Figure 3.20: Fitting procedure

When the fitting process is finished, you will get the fitting results as demonstrated in Fig. [3.21, 3.22].
At the bottom of the page, see Fig. 3.22, you can select what to do with the `fitting result`. The options include: `SAVE AS CARD`, `EXPORT AS FILE`, `GENERATE TRACE`, `GET RANDOM NUMBER GENERATOR`, `DISCARD`, `BACK`.

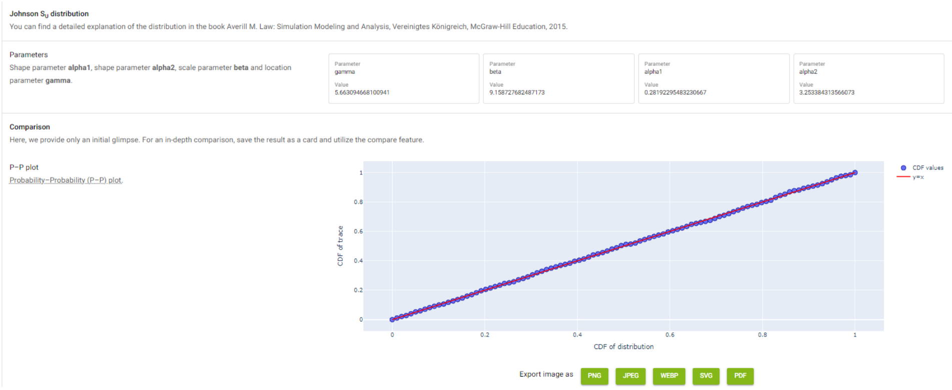


Figure 3.21: Fitting result

Figure 3.22: Fitting procedure

3.4 Compare Distribution and Trace

Now let's compare the distribution card and the trace card we created earlier by selecting `COMPARE SELECTED CARDS` as shown in Fig. 3.23.
We get redirected to the `Compare Page`, see Fig. 3.24.

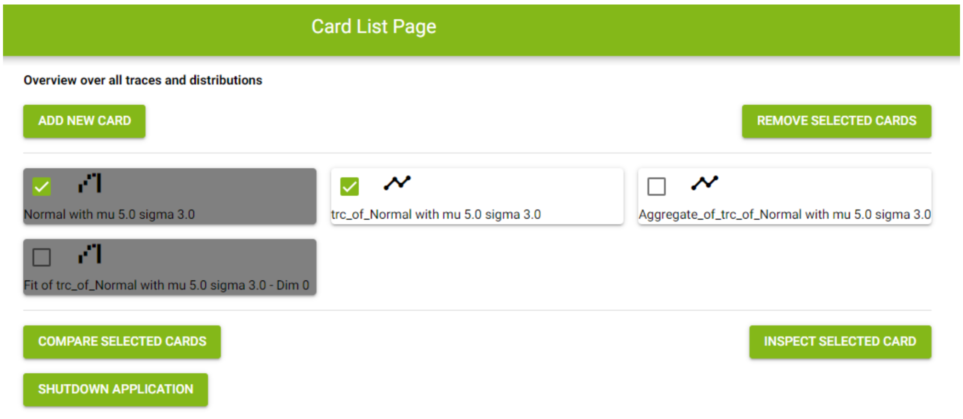


Figure 3.23: Compare distribution and trace card

Here we get the following comparisons:

1. `Basic information`. This section lists the essential details of the distribution and trace cards, as shown in Fig. 3.25.

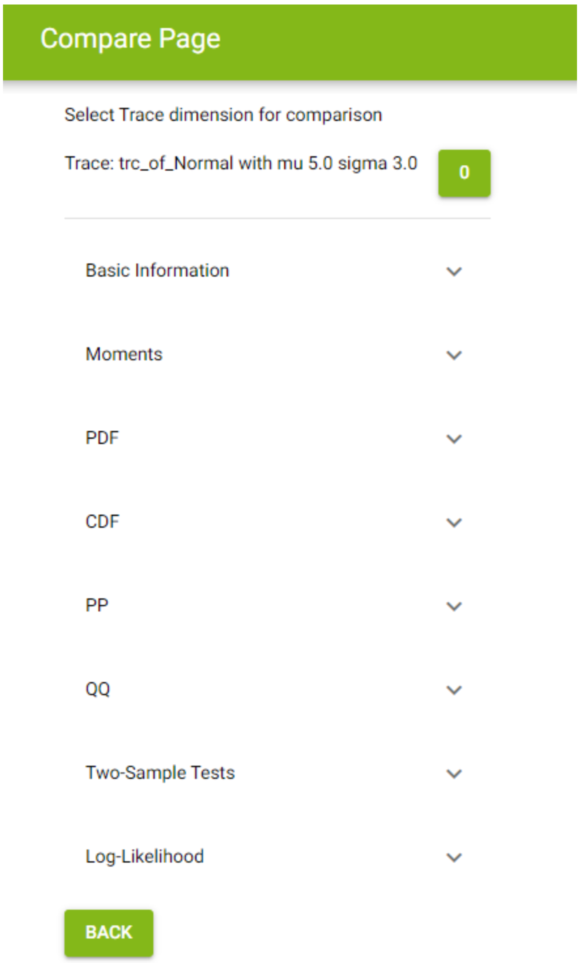
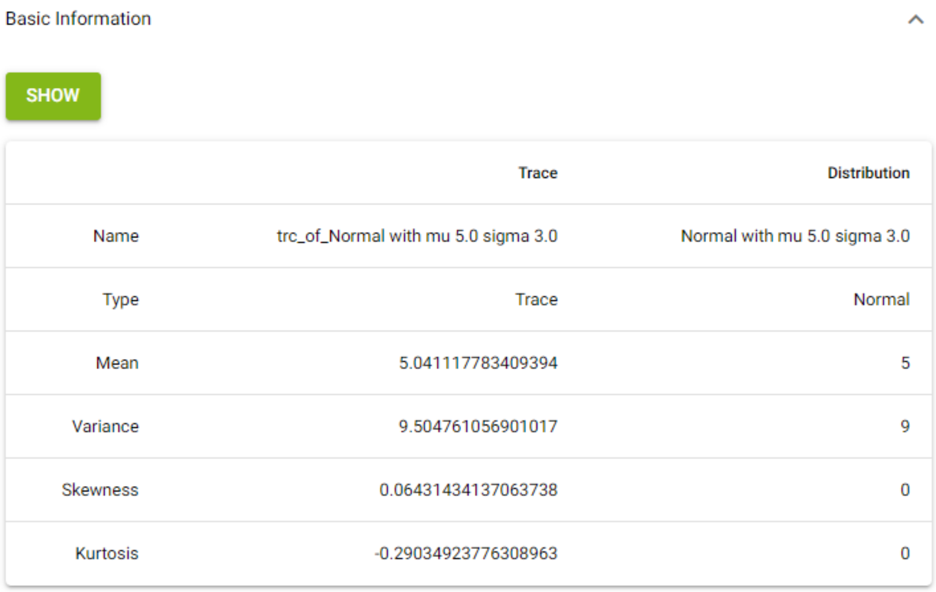


Figure 3.24: Compare page



Basic Information

SHOW

	Trace	Distribution
Name	trc_of_Normal with mu 5.0 sigma 3.0	Normal with mu 5.0 sigma 3.0
Type	Trace	Normal
Mean	5.041117783409394	5
Variance	9.504761056901017	9
Skewness	0.06431434137063738	0
Kurtosis	-0.29034923776308963	0

Figure 3.25: Basic information

2. `Moments`: A moment plot is generated, which compares the (ratio of) moments of the `trace` and the `fitted distribution`, as illustrated in Fig. 3.26.

   The plot visually compares how closely the moments of the trace align with the moments of the fitted distribution. The closer the lines are, the better the distribution fits the trace data across different moments.

Figure 3.26: Moments

3. `PDF`. This plot compares the PDF of the `trace` data and the `fitted distribution`, as shown in Fig. 3.27.

   The plot visually illustrates how well the fitted distribution approximates the underlying data. Here the fit appears reasonable, as the red curve aligns well with the blue bars, particularly around the center, where most of the data is concentrated.

Figure 3.27: PDF

4. `CDF`: This plot compares the CDF of the `trace` data and the `fitted distribution`, as shown in Fig. 3.28.

   The closer the lines of the `trace` and `fitted distribution` match, the better the fitted distribution approximates the cumulative behavior of the trace data. In this case, the two lines are very close, indicating that the fitted distribution provides a good approximation of the trace data's cumulative probabilities.

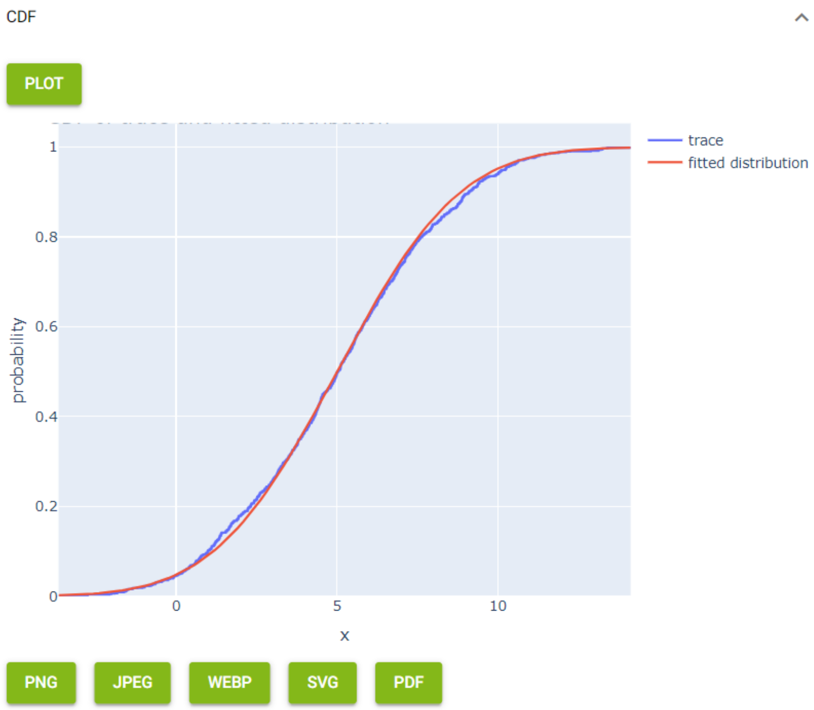


Figure 3.28: CDF

5. `PP` : The PP plot compares the CDF of the trace (on the y-axis) with the CDF of the fitted distribution (on the x-axis), as shown in Fig. 3.29.

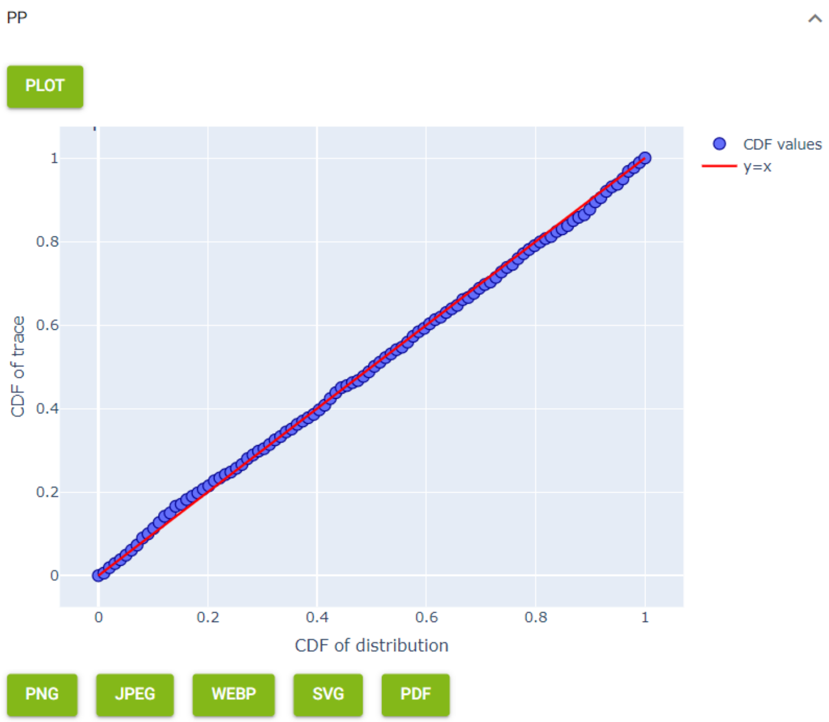


Figure 3.29: PP-plot

6. QQ: The QQ plot compares the quantiles of the trace (on the y-axis) with the quantiles of the distribution (on the x-axis), see Fig. 3.30.

Figure 3.30: QQ-plot

7. `Two-Sample Tests`: The results of two-sample statistical tests (`kstest`, `chisquared`, and `cramervonmises`) comparing the trace and fitted distribution are displayed here, as illustrated in Fig. 3.31.

8. `Log Likelihood`: The log-likelihood indicates how well the fitted distribution explains the trace data, as shown in Fig. 3.31.

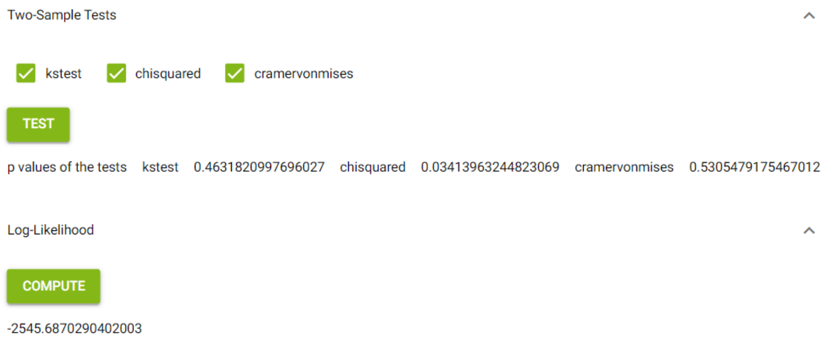


Figure 3.31: Two-Sample Tests and Log-Likelihood

3.5 Compare Trace and Fitted Distribution

Now we will compare the trace card with the fitted distribution card we created earlier by selecting `COMPARE SELECTED CARDS` as shown in Fig. 3.32.

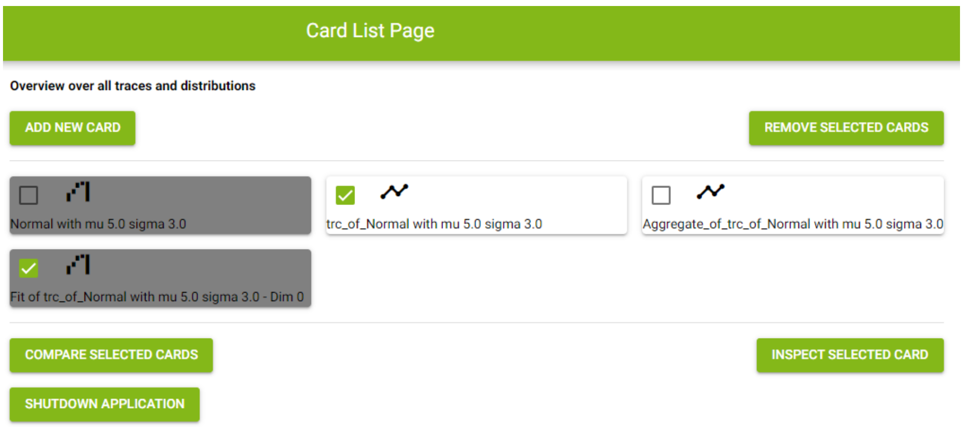


Figure 3.32: Compare trace and fitted distribution card

We get redirected to the `Compare Page`, see Fig. 3.33.

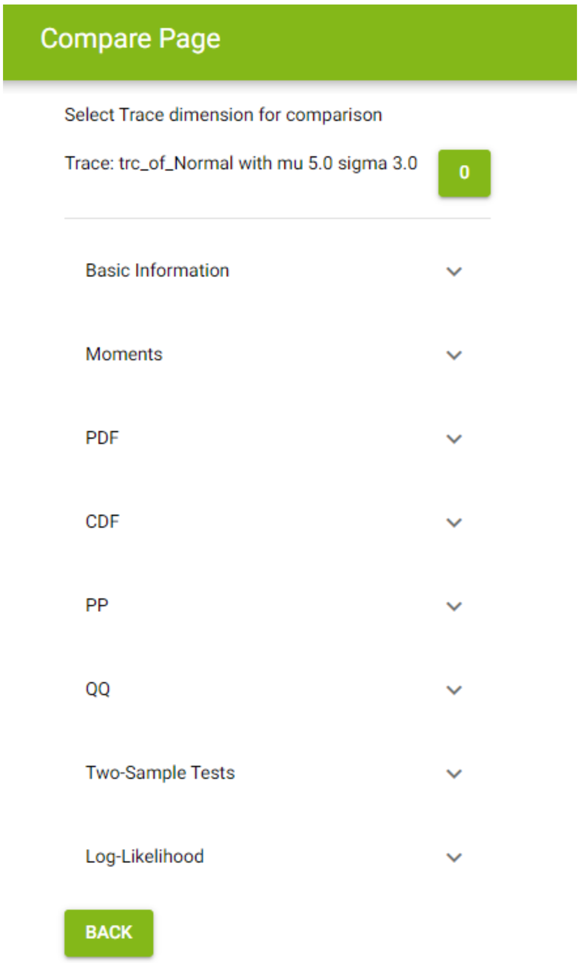


Figure 3.33: Compare page

Here we get the following comparisons:

1. `Basic information`. The basic details of the fitted distribution and trace cards are presented in Fig. 3.34.

Basic Information

SHOW

	Trace	Distribution
Name	trc_of_Normal with mu 5.0 sigma 3.0	Fit of trc_of_Normal with mu 5.0 sigma 3.0 - Dim 0
Type	Trace	JohnsonSU
Mean	5.041117783409394	5.040650079452533
Variance	9.504761056901017	9.506321573322863
Skewness	0.06431434137063738	0.07683330696099726
Kurtosis	-0.29034923776308963	0.011316731815497416

Figure 3.34: Basic information

2. `Moments`: A moment plot is generated here to compare the (ratio of) moments of the `trace` and the `fitted distribution`, as shown in Fig. 3.35.

   Up to around the 8th moment, the lines follow a similar trajectory, indicating a reasonable match between the trace and the fitted distribution. However, from the 9th and 10th moments, the red line shows a large spike, indicating that the fitted distribution differs significantly from the trace.

Figure 3.35: Moments

3. `PDF`. This plot compares the PDF of the `trace` data and the `fitted distribution`, as illustrated in Fig. 3.36.

   In this case, the red line (fitted distribution) follows the general shape of the blue bars (trace data) reasonably well.

Figure 3.36: PDF

4. `CDF`: This plot compares the CDF of the `trace` data and the `fitted distribution`, as shown in Fig. 3.37.

   In this plot, the red line (fitted distribution) closely follows the blue line (trace data), indicating that the fitted distribution provides a good approximation of the cumulative behavior of the trace data. Small deviations are visible, but overall, the CDFs align well.

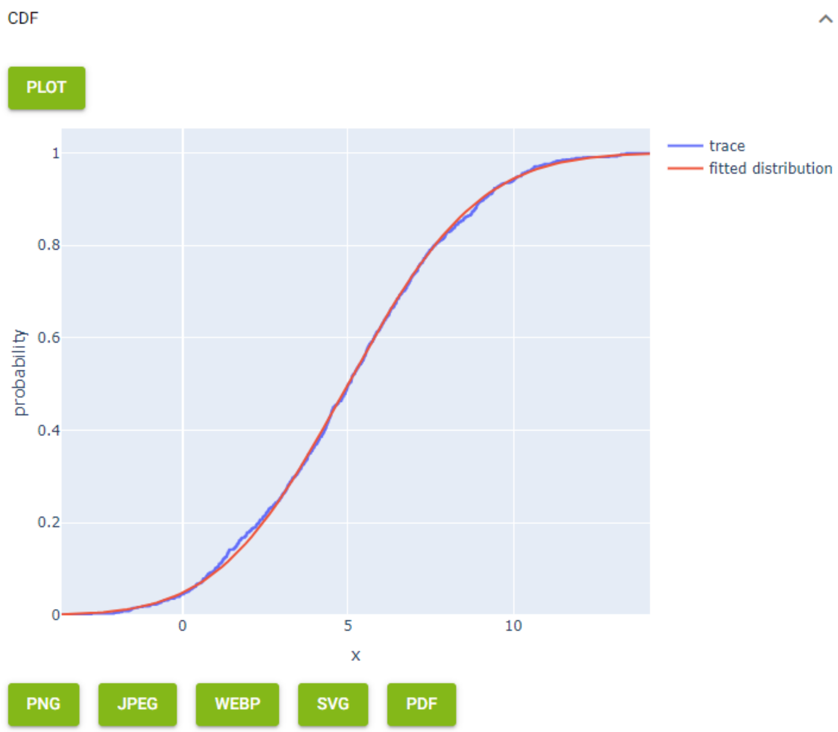


Figure 3.37: CDF

5. `PP` : The PP-plot compares the CDF of the trace (on the y-axis) with the CDF of the distribution (on the x-axis), as shown in Fig. 3.38.

   The fact that the blue points closely follow the red line suggests that the CDFs of the trace and the fitted distribution are very similar, indicating a good fit.

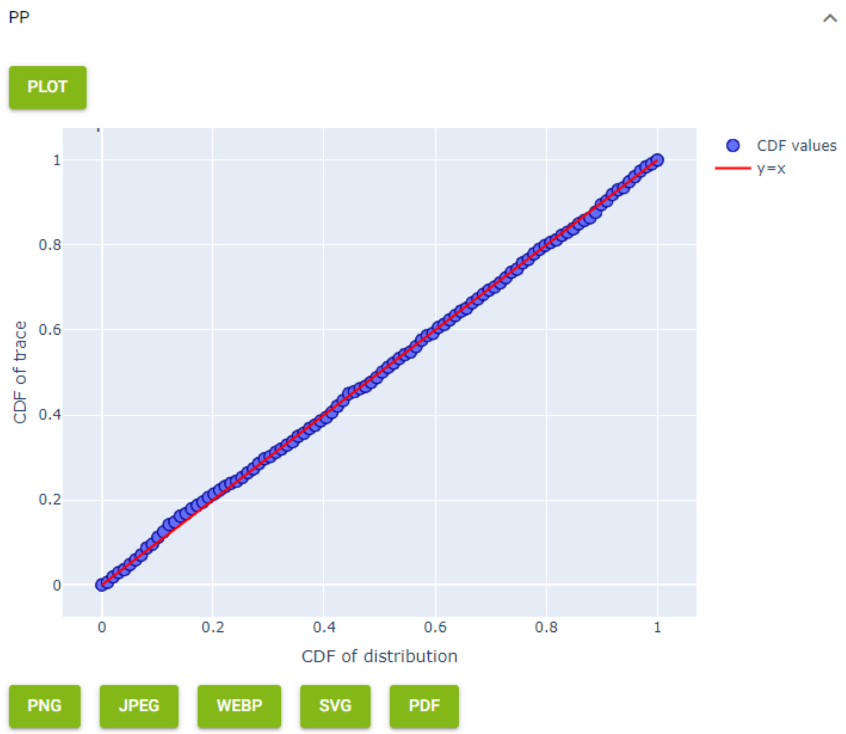


Figure 3.38: PP-plot

6. `QQ`: The QQ-plot compares the quantiles of the trace (on the y-axis) with the quantiles of the distribution (on the x-axis), as shown in Fig. 3.39.

Figure 3.39: QQ-plot

7. `Two-Sample Tests`: This section presents the results of two-sample statistical tests (`kstest`, `chisquared`, and `cramervonmises`) that compare the trace and the distribution, as shown in Fig. 3.40.

8. `Log Likelihood`: The log-likelihood indicates how well the fitted distribution explains the trace data, as shown in Fig. 3.40.

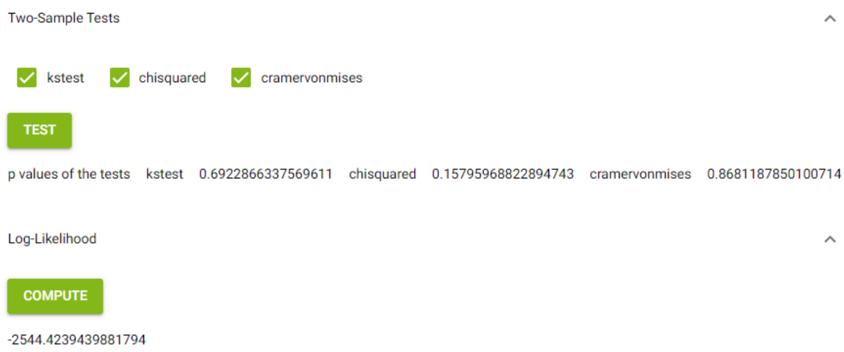


Figure 3.40: Two-Sample Tests and Log-Likelihood

4 Working with ASMo

In this section, we will explore how to effectively use ASMo for modeling and analyzing stochastic input models. We will provide a comprehensive explanation of each option available in ASMo, detailing its functionality and application. By the end of this section, you will have a clear understanding of how to leverage these tools to enhance your modeling and analytical capabilities.
The ASMo program offers three tabs: the `Home` tab, the `Card List` tab, and the `Download` tab.

4.1 Homepage

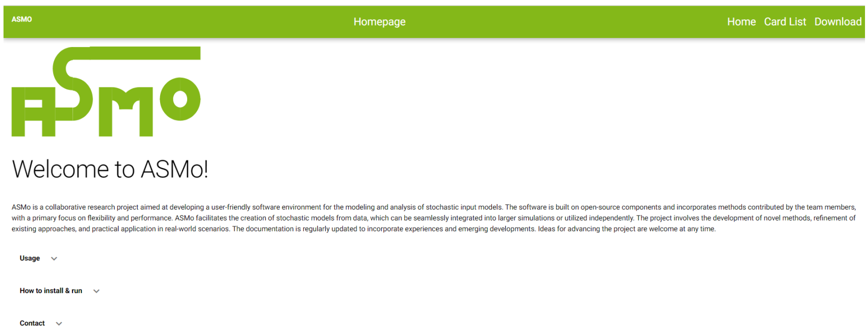


Figure 4.1: Homepage

On the homepage of ASMo, Fig. 4.1, you will find a brief description of ASMo, along with instructions on its `Usage`, a manual on `How to install & run` ASMo, and `Contact` information.

4.2 Card List

On the `Card List Page` , Fig. 4.2, you will find the following options:

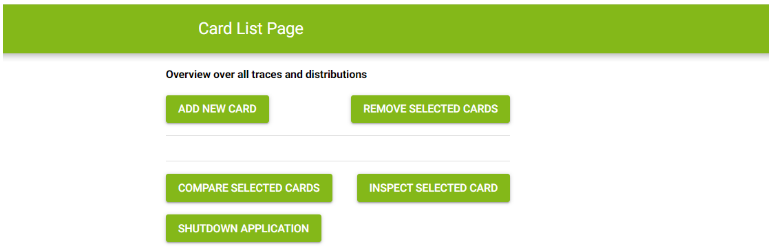


Figure 4.2: Card list

4.2.1 Add new Card

- The option `ADD NEW CARD` adds a new card of trace or distribution.

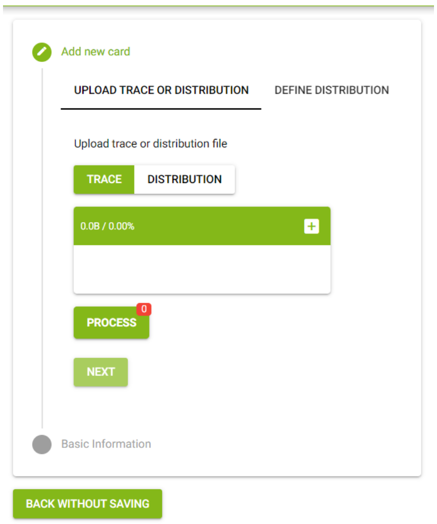


Figure 4.3: Upload trace or distribution

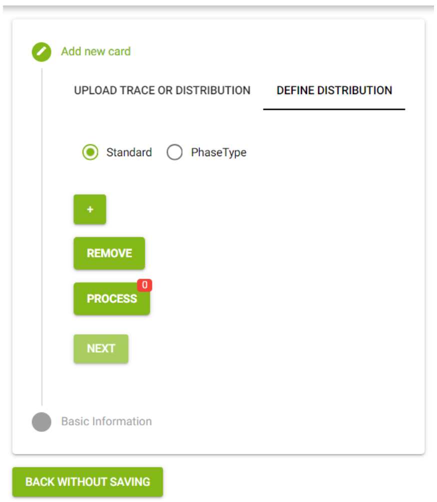


Figure 4.4: Define distribution

- When you select `ADD NEW CARD`, a new menu with options appears, as shown in Fig. [4.3, 4.4].
- This section offers you two possibilities:

1. `UPLOAD TRACE OR DISTRIBUTION`, as shown in Fig. 4.3:

- You can choose whether to
  `Upload trace or distribution file.`
- Next, select the symbol + to choose a distribution or trace file from your computer.
- After that, proceed to upload the file.
- Once you have uploaded a distribution or trace file in the correct format, you can click on `PROCESS` followed by `NEXT`, to create the card.
  Alternatively, you can select `BACK WITHOUT SAVING`, which will return you to the
  `Card List Page`, as shown in Fig. 4.2.

2. `DEFINE DISTRIBUTION` , as shown in Fig. 4.4:
   In this section, you can choose between `Standard` and `PhaseType`.
   Let's start by selecting `Standard`, which corresponds to a standard distribution.
   - First, click the + symbol. This will display the following elements, as shown in Fig. 4.5. Here, you can select an option to choose from various standard probability distributions for defining a distribution, including `Beta`, `Erlang`, `Exponential`, `Gamma`, and others. For each distribution type selected, you must also specify the parameters; for example, for the `Beta` distribution, you need to specify `alpha1: shape parameter` and `alpha2: shape parameter`.
   - You can click the + symbol multiple times to define a mixture model by combining different distributions, as illustrated in Fig. 4.6.
     When defining a mixture model, you will also encounter the option to input the weight vector, denoted as
     `Please type vector pi in format [..., ...]` which specifies the contribution of each distribution to the overall mixture.
   - To remove a defined distribution, check the box at the bottom left of the distribution and click `REMOVE`. You can select multiple distributions to remove at once.
   - Clicking `BACK WITHOUT SAVING` will take you back to the
     `Card List Page`, see Fig. 4.2.
   - When you click `PROCESS` or `NEXT`, you will be redirected to a new page, as shown in Fig. [4.7, 4.8].
   - If you choose `PREVIOUS STEP` in Fig. 4.8, you will return to the previous page, Fig. 4.7.
   - Similarly, clicking `BACK WITHOUT SAVING` in Fig. 4.8 will take you back to the `Card List Page`, Fig. 4.2.

- Finally, when you click `CREATE CARD`, a new card will be created.

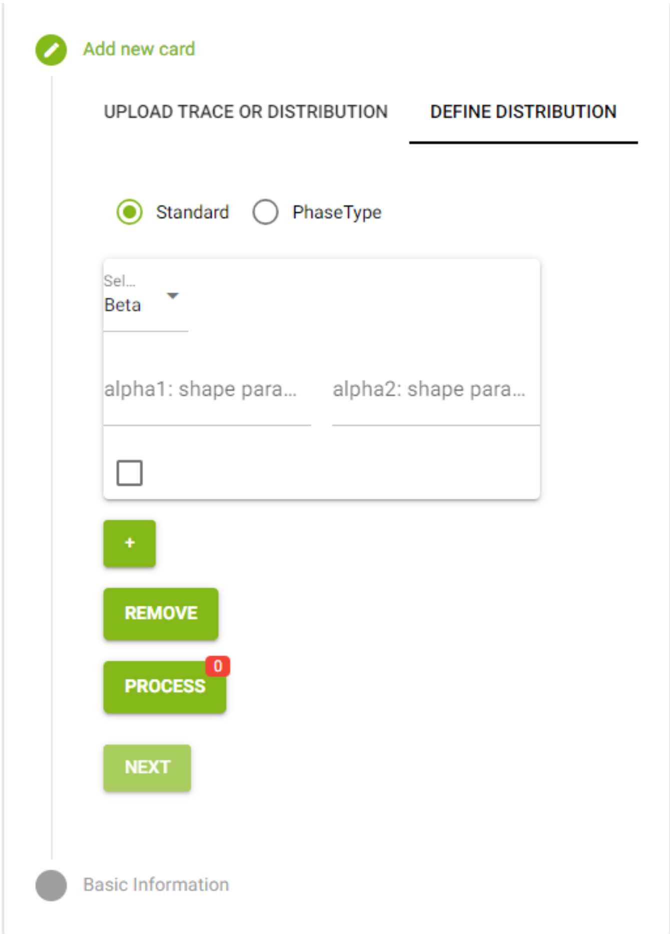


Figure 4.5: Define distribution

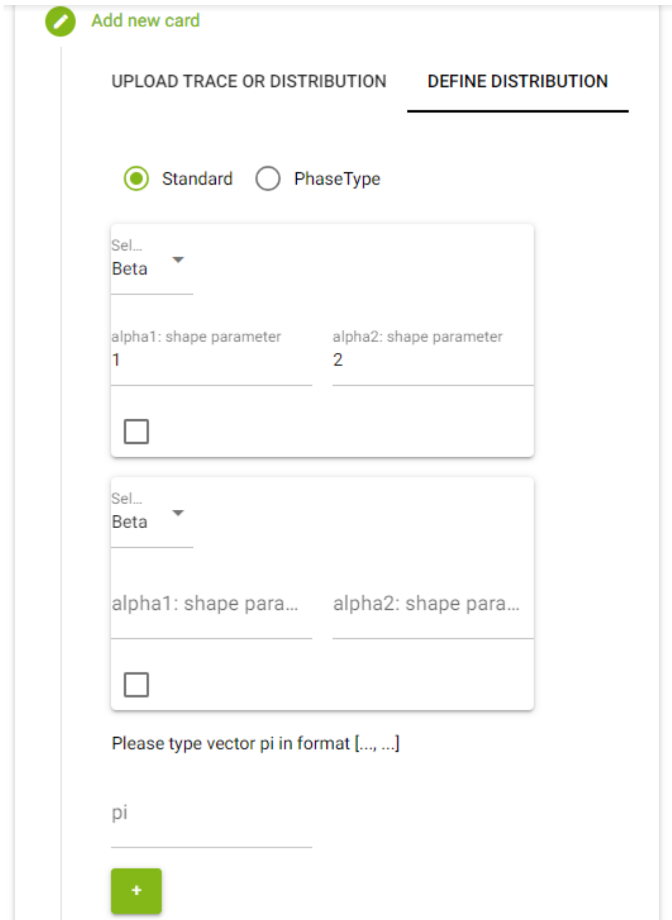


Figure 4.6: Define distribution

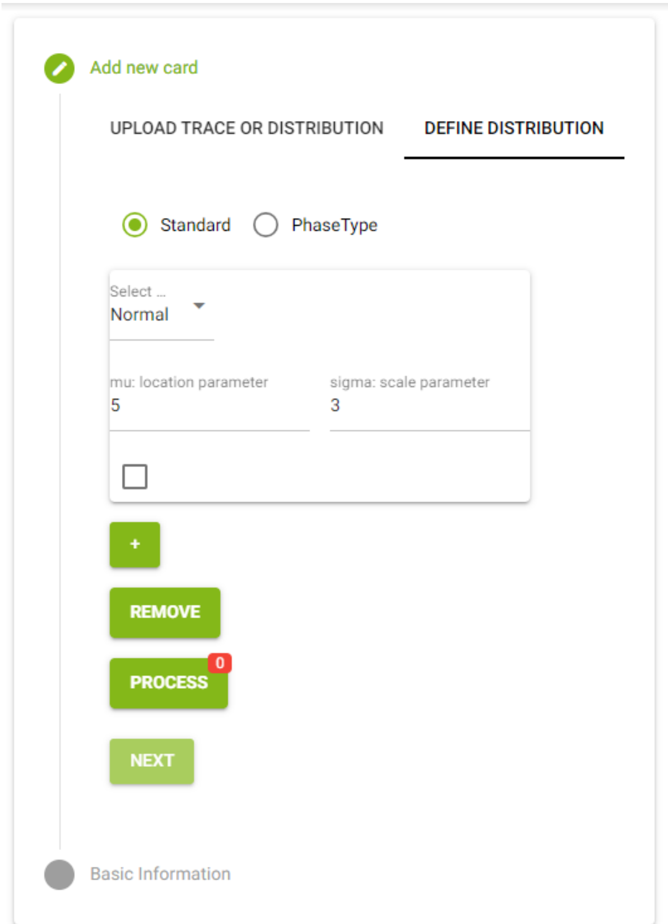


Figure 4.7: Define distribution

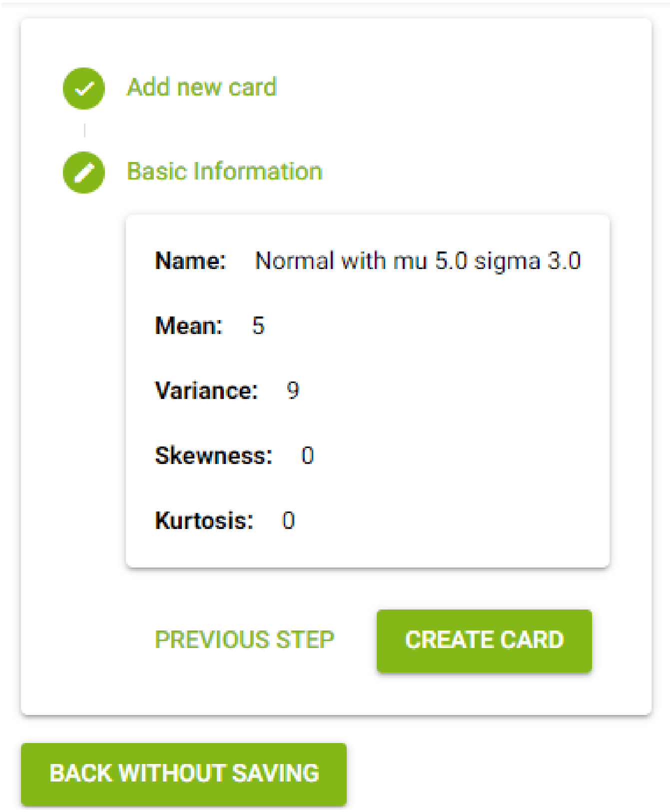


Figure 4.8: Define distribution

4.2.2 Remove selected Cards

This option allows you to remove selected cards, as illustrated in Fig. 4.9. To do this, select the cards you wish to remove by checking the boxes in the top right corner of each card. Then, click `REMOVE SELECTED CARDS`. You can remove multiple cards simultaneously.

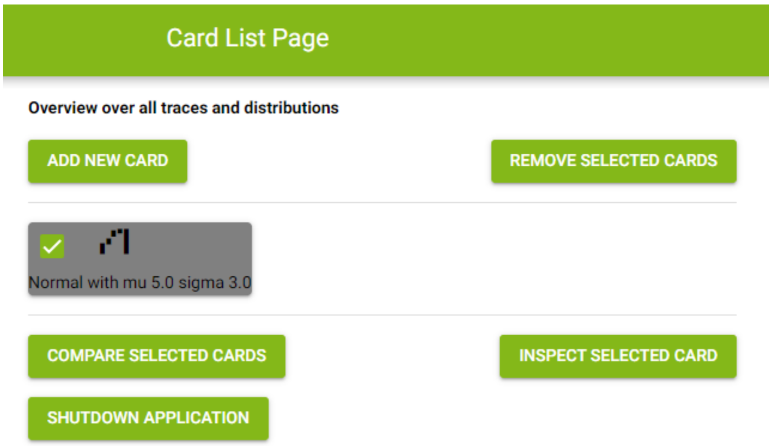


Figure 4.9: Remove selected cards

4.2.3 Compare selected Cards

This option is used to `compare selected cards`. First, select the cards you want to compare by checking the boxes in the top right corner of each card, and then click `COMPARE SELECTED CARDS`, as shown in Fig. 4.10.
You will be redirected to the `Compare Page`, as shown in Fig. 4.11.
We now have the following comparisons available:

1. `Basic information` : This section presents the essential details of the distribution and trace cards, as shown in Fig. 4.12.

   The following information is listed in the table for both `trace` and `distribution`:the `Name` of the `Trace` and `Distribution`, `Type`, which indicates the nature of the data in each column, `Mean`, `Variance`, `Skewness`, `Kurtosis`.

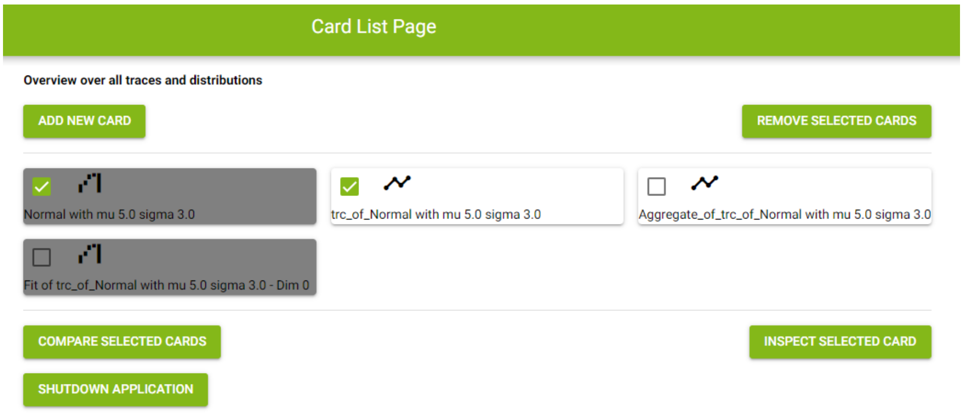


Figure 4.10: Compare distribution and trace card

2. `Moments`: This section generates a moment plot that compares the (ratio of) moments of the `trace` (in blue) and the `fitted distribution` (in red), as shown in Fig. 4.13. The plot visually illustrates how closely the moments of the trace align with those of the fitted distribution. The closer the lines are to each other, the better the fitted distribution corresponds to the trace data across various moments.

   You can save the `moment` plot in one of the given formats: `PNG`, `JPEG`, `WEBP`, `SVG`, `PDF`.

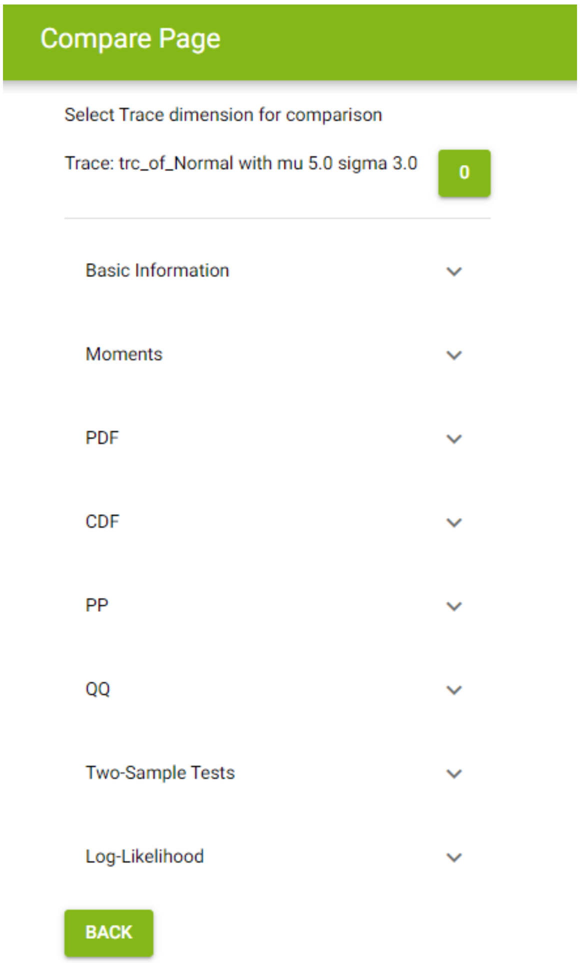


Figure 4.11: Compare page

Basic Information

SHOW

	Trace	Distribution
Name	trc_of_Normal with mu 5.0 sigma 3.0	Normal with mu 5.0 sigma 3.0
Type	Trace	Normal
Mean	5.041117783409394	5
Variance	9.504761056901017	9
Skewness	0.06431434137063738	0
Kurtosis	-0.29034923776308963	0

Figure 4.12: Basic information

Figure 4.13: Moments

3. `PDF`. This plot compares the PDF of the `trace` data with the `fitted distribution`, as shown in Fig. 4.14.

   The plot visually illustrates how well the fitted distribution approximates the underlying data. Here the fit appears reasonable, as the red curve aligns well with the blue bars, particularly around the center, where most of the data is concentrated.

   You can export the `PDF` plot in one of the given formats: `PNG`, `JPEG`, `WEBP`, `SVG`, `PDF`.

Figure 4.14: PDF

4. `CDF`: This plot compares the CDF of the `trace` data and the `fitted distribution`, as shown in Fig. 4.15.

   The closer the lines of the `trace` and `fitted distribution` match, the better the fitted distribution approximates the cumulative behavior of the trace data. In this case, the two lines are very close, indicating that the fitted distribution provides a good approximation of the trace data's cumulative probabilities.

   You can export the `CDF` plot in one of the given formats: `PNG`, `JPEG`, `WEBP`, `SVG`, `PDF`.

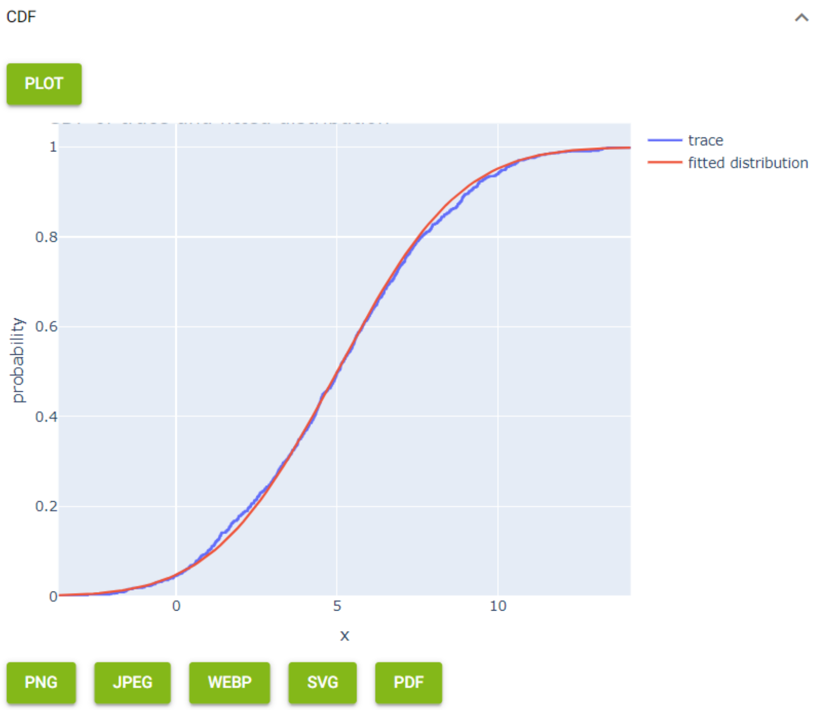


Figure 4.15: CDF

5. PP : The PP plot compares the CDF of the trace (on the y-axis) with the CDF of the fitted distribution (on the x-axis), as shown in Fig. 4.16.

   You can save the PP plot in one of the given formats: PNG, JPEG, WEBP, SVG, PDF.

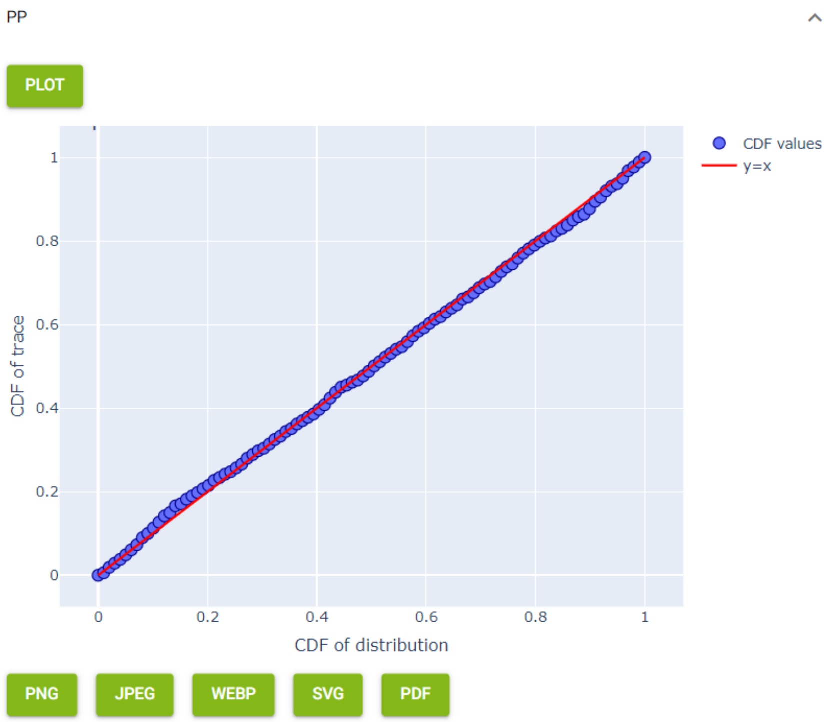


Figure 4.16: PP-plot

6. QQ: The QQ plot compares the quantiles of the trace (on the y-axis) with the quantiles of the distribution (on the x-axis), as shown in Fig. 4.17.

   You can save the QQ plot in one of the given formats: PNG, JPEG, WEBP, SVG, PDF.

Figure 4.17: QQ-plot

7. Two-Sample Tests: This section presents the results of two-sample statistical tests (kstest, chisquared, and cramervonmises) that compare the trace with the fitted distribution, as illustrated in Fig. 4.18.

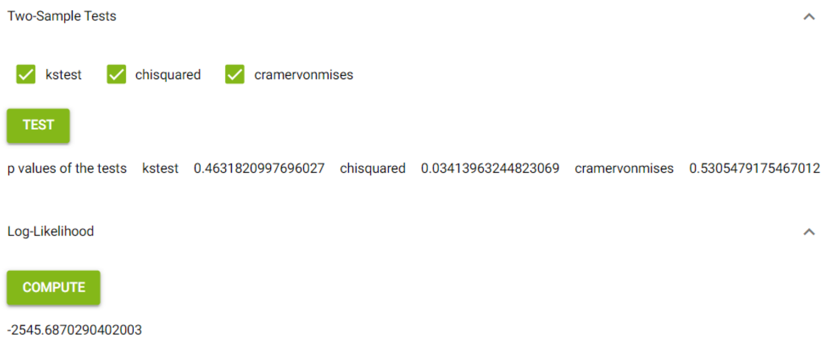


Figure 4.18: Two-Sample Tests and Log-Likelihood

8. Log Likelihood: The log-likelihood indicates how well the fitted distribution explains the trace data, as shown in Fig. 4.18.

9. BACK brings you back to the Card List Page.

4.2.4 Inspect selected Card

This option is used to INSPECT SELECTED CARD as shown in Fig. 4.19.
If you are inspecting a distribution card, you have the following options:

- Basic information: As shown in Fig. 4.20, this option displays all the essential details of your distribution card. It includes the following: Name, which is the assigned name of the distribution card; Type, indicating the type of distribution (e.g., normal in our case); Description, providing a brief overview of the distribution and additional statistical details such as Parameters, Mean, Variance, Skewness, Kurtosis.

- Set name and description: As shown in Fig. 4.21, you can assign a new name and description to the selected card you are inspecting. By clicking SET NEW VALUES, the changes will be saved.

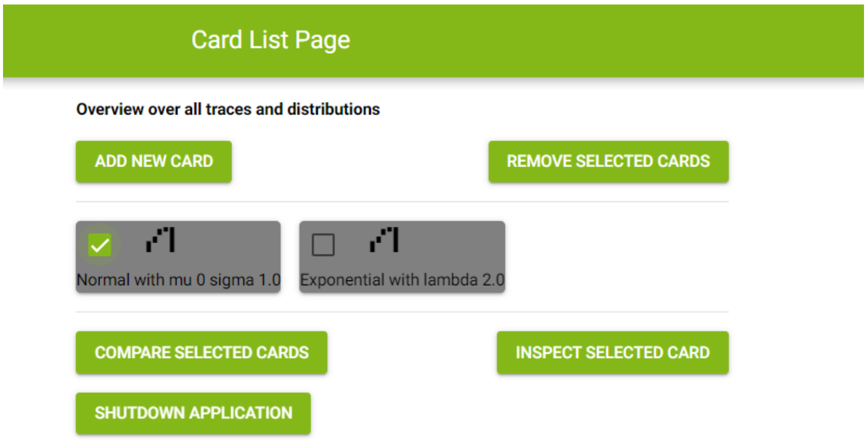


Figure 4.19: Inspect selected card

Basic information

Field	Value
Name	Normal Distribution
Type	Normal
Description	This is a normal distribution with a mean of 5 and a standard deviation of 3.
Parameters	{'mu': 5, 'sigma': 3}
Mean	5.0
Variance	9.0
Skewness	0.0
Kurtosis	0.0

Figure 4.20: Basic information

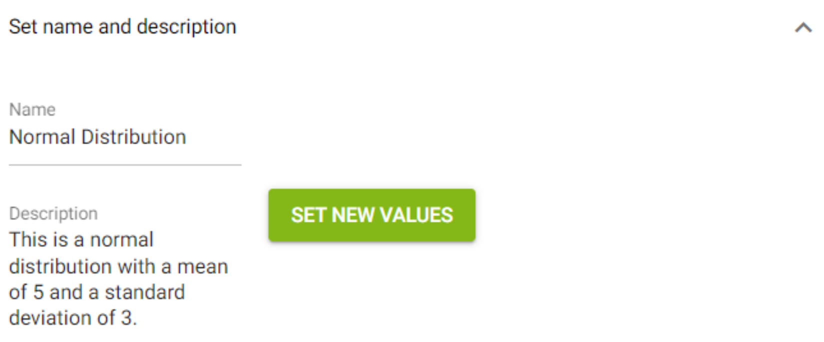


Figure 4.21: Set name and description

- CDF: As illustrated in Fig. 4.22, you can generate the CDF of the distribution.

  Additionally, you can also download the CDF plot in one of the following formats: PNG, JPEG, WEBP, SVG or PDF.

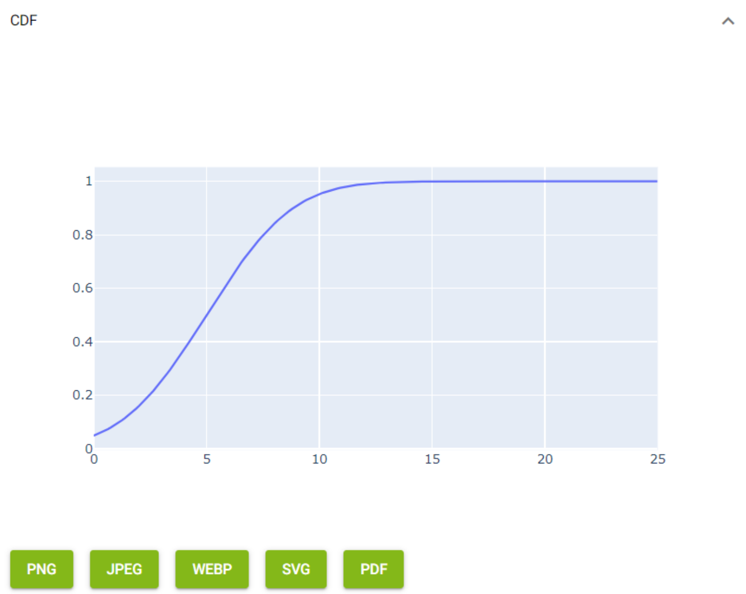


Figure 4.22: CDF of distribution

- PDF: As shown in Fig. 4.23, you can generate the PDF of the distribution.

  You can also download the PDF plot in one of the following formats: PNG, JPEG, WEBP, SVG or PDF.

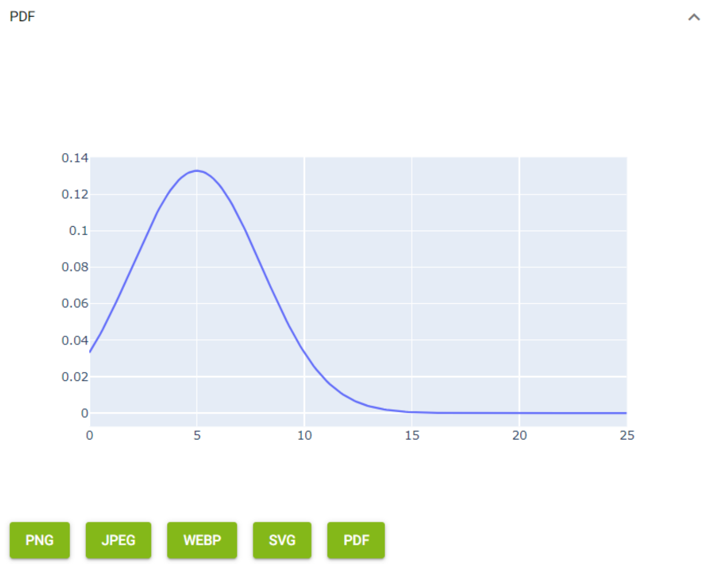


Figure 4.23: PDF of distribution

- `Generate trace`: As shown in Fig. 4.24, you can `generate a trace` by giving the `number of elements`, which refers to how many random data points (or elements) you want to generate from the specified distribution and then clicking `generate`.
- `Export`: As shown in Fig. 4.25, You can export the distribution card in `yaml`, c++ or `python` format. In order to export the distribution card, you have to choose the `format` first and then click `download`.
- `Back`: This option redirects you to the previous page: `Card List Page`, Fig. 4.9.

If you are inspecting a trace card, you have the following options:

- `Basic information`: As shown in Fig. 4.26, the `basic information` of the trace includes the following details: `Name`, which is the name of the trace; `Type`, refers to the type of the data, which in our case is a `Trace`;

  `Source`, refers to the origin of data used to generate the trace, which in our case is None, because the trace was created internally;

  `Description`, which is the description of the trace;

  `Dimension`, refers to the number of variables or features in the data, in our case the dimension is 1, which mean that the data is one-dimensional, a dimension greater than 1 would indicate a multi-dimensional dataset and

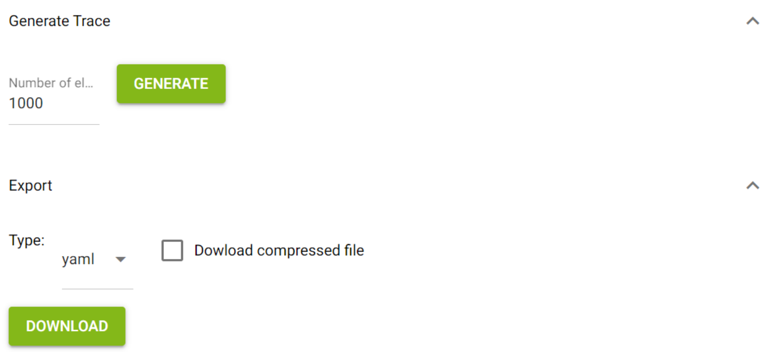


Figure 4.24: Generate trace

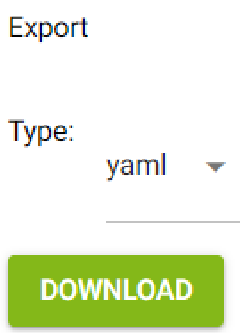


Figure 4.25: Export distribution

`No. of Entries`, refers to the total number of data points or elements in the trace or dataset, in our case 1000 means that the trace consists of 1000 individual data points.

Basic information

Field	Value
Name	trc_of_Normal Distribution
Type	Trace
Source	None
Description	None
Dimension	1
No. of Entries	1000

Figure 4.26: Basic information

- `Set Name and Description`: As demonstrated in Fig. 4.27, you can set a new name and description for your card. To save the changes click `SET NEW VALUES`.

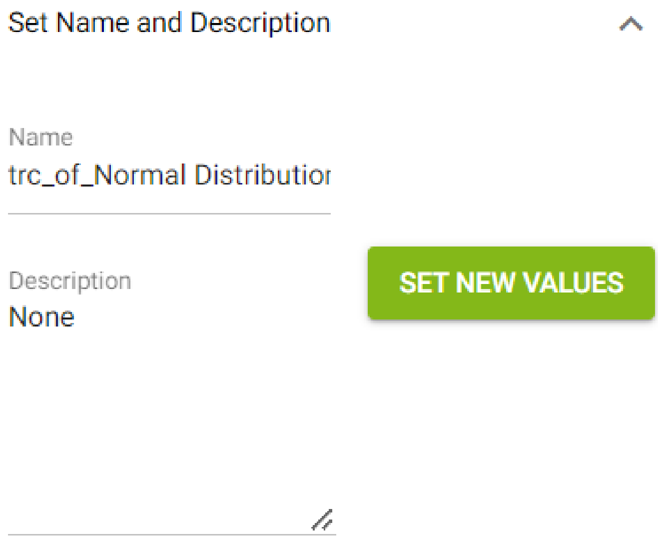
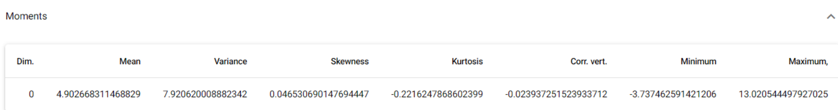


Figure 4.27: Set name and description

- `Moments`: As shown in Fig. 4.28, the option `Moments` gives information about the dimensions and parameter values like `Mean`, `Variance`, `Skewness`, `Kurtosis`, `Corr. vert.`, `Minimum` and `Maximum`.


Moments

Dim.	Mean	Variance	Skewness	Kurtosis	Corr. vert.	Minimum	Maximum,
0	4.902668311468829	7.920620008882342	0.046530690147694447	-0.2216247868602399	-0.023937251523933712	-3.737462591421206	13.020544497927025

Figure 4.28: Moments

- `Values`: As illustrated in Fig. 4.29, `Dim` (short for Dimension) refers to the number of components or features associated with each entry or data point.

  For a one-dimensional trace (like `Dim. 0`), each entry consists of a single scalar value.

  If the trace or dataset were multi-dimensional (e.g., Dim. 1, Dim. 2, etc.), each entry would consist of multiple values.

You can also download the Value plot by clicking `PLOT VALUES` in one of the following formats: `PNG`, `JPEG`, `WEBP`, `SVG` or `PDF`.

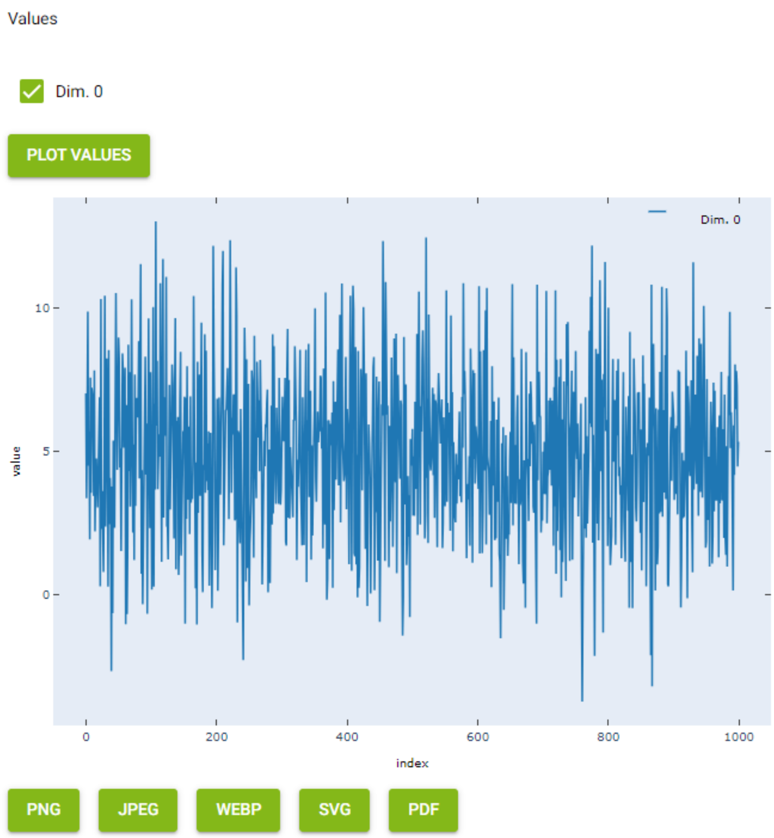


Figure 4.29: Values

- `CDF`: This section allows you to generate the CDF of the trace, see Fig. 4.30.

  `Dim` refers to the dimensionality of the data. `Dim. 0` stands for One-dimensional, scalar values. Higher dimensions would refer to more complex data with multiple features per data point.

  You can also download the CDF plot by clicking `PLOT CDF` in one of the following formats: `PNG`, `JPEG`, `WEBP`, `SVG` or `PDF`.

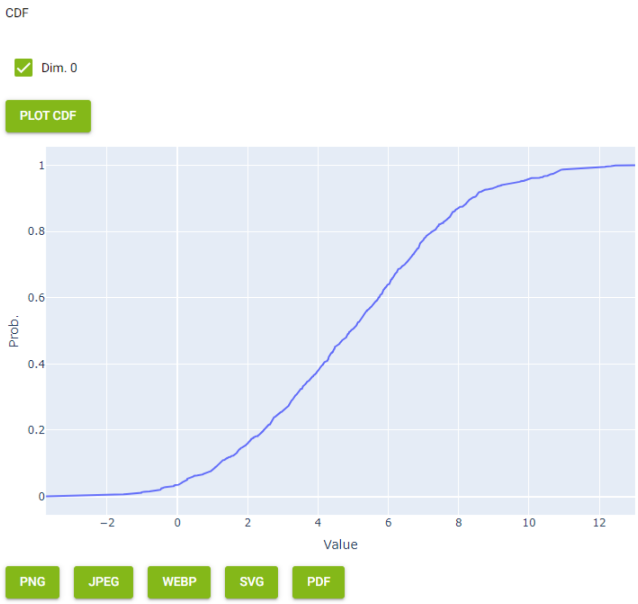


Figure 4.30: CDF

- PDF As shown in Fig. 4.31, here you can generate the PDF of the trace.

  Dim refers to the dimensionality of the data. Dim. 0 stands for One-dimensional, scalar values. Higher dimensions would refer to more complex data with multiple features per data point.

  The number of bins in the histogram refers to the number of intervals into which the range of data is divided. Each bin represents a segment of the data's range, and the height of each bar in the histogram reflects the frequency or density of the data points that fall within that specific range.

  You can also download the PDF plot by clicking PLOT PDF in one of the following formats: PNG, JPEG, WEBP, SVG or PDF.

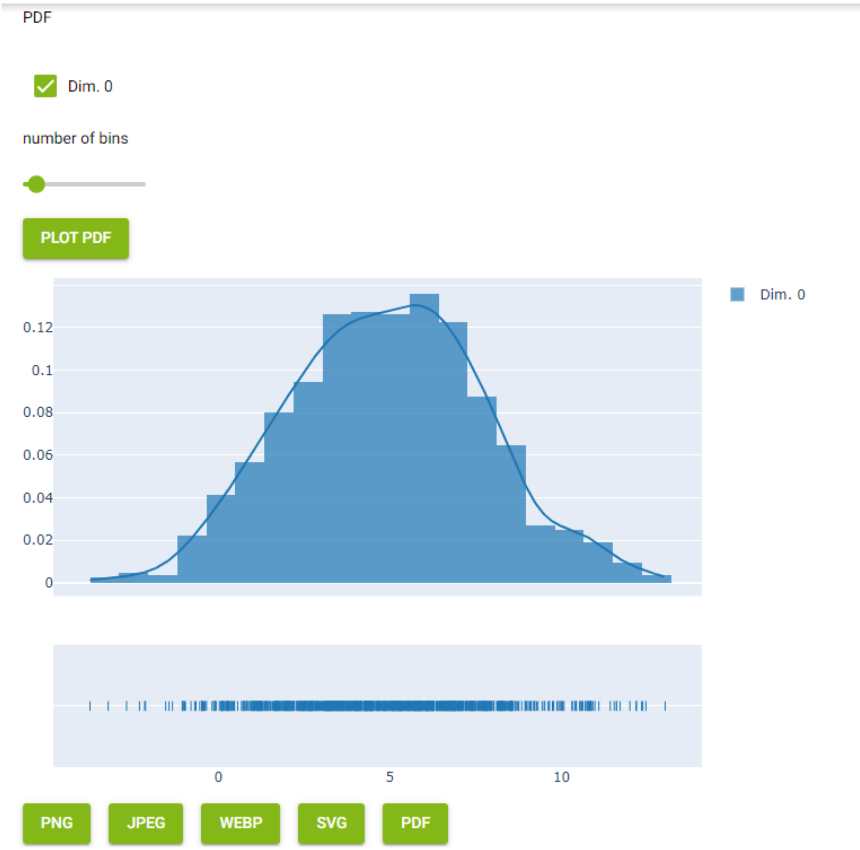


Figure 4.31: PDF

- `Lag k autocorrelation`: As shown in Fig. 4.32, you can inspect `lag k autocorrelation` by choosing the `lag k autocorrelation` and then `plotting lag k autocorrelation`.

  `Dim` refers to the dimensionality of the data. `Dim. 0` stands for One-dimensional, scalar values. Higher dimensions would refer to more complex data with multiple features per data point.

  The `lag k autocorrelation` measures the correlation between a time series and a lagged version of itself, specifically with a lag of k steps.

  The lag plot can be downloaded in one of the following formats: `PNG`, `JPEG`, `WEBP`, `SVG`, `PDF`.

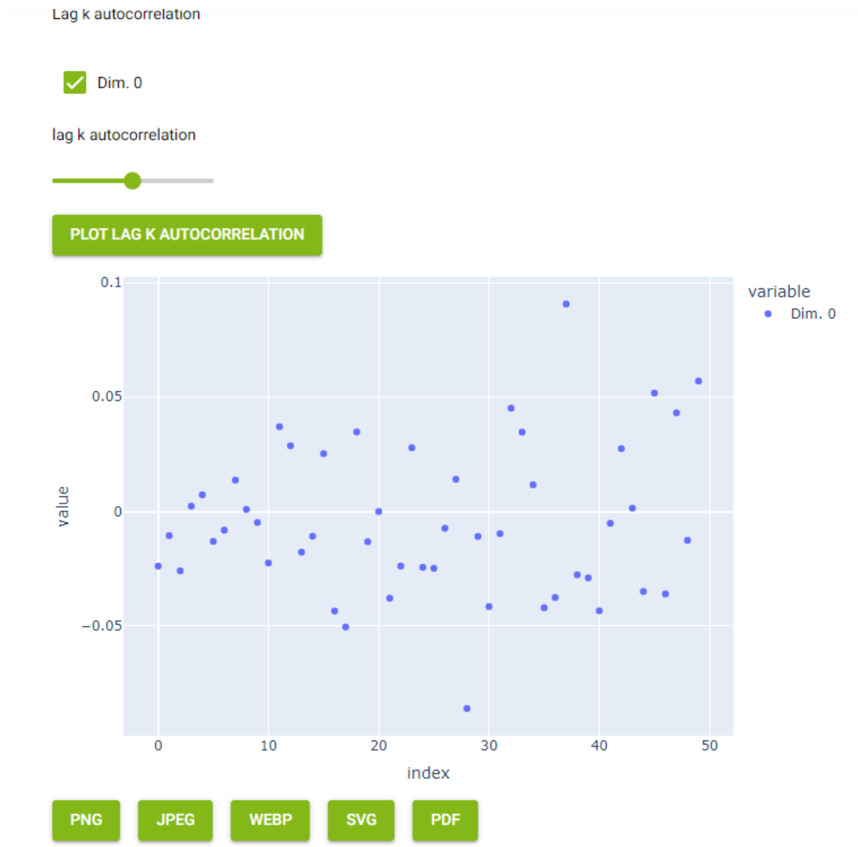


Figure 4.32: lag k autocorrelation

- `Change trace properties`: This section allows you to modify the properties of the trace, see Fig. 4.33.

  `Select dimensions for modification` allows you to choose which dimension of the trace data you want to modify. In cases where the trace has multiple dimensions, you can specify which one to apply the modifications to.

  `Add value`: Adds a constant value to all the elements in the selected dimension of the trace.

  `Substitute zero by`: Replaces any zero values in the trace with the specified value.

  `Cut values below`: Removes any values in the trace that are lower than the specified cut-off value.

  `Cut values above`: Removes any values in the trace that are higher than the specified cut-off value.

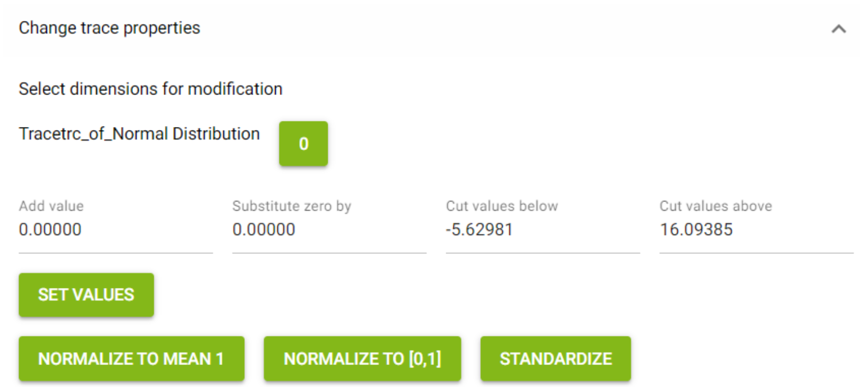


Figure 4.33: Change trace properties

- `Select and copy`: The `Select and Copy` feature allows you to create a new trace by copying elements from an existing trace, see Fig. 4.34. You can `select dimensions to copy` and you can `select elements to copy`.

  `Dim` refers to the dimensionality of the data. The option `elements to copy` allows you to choose a range of elements (values) from the trace that you want to copy.

Figure 4.34: Select and copy

- `Aggregate one dimension`: Here you can `select dimension to aggregate` and you can choose the number of `bins`, as shown in Fig. 4.35. When you click `aggregate`, a new card will be created.

  The `Select dimension to aggregate` option allows you to choose which dimension of the data you want to aggregate.

  The number of `bins` defines how the data from the selected dimension will be grouped into intervals.

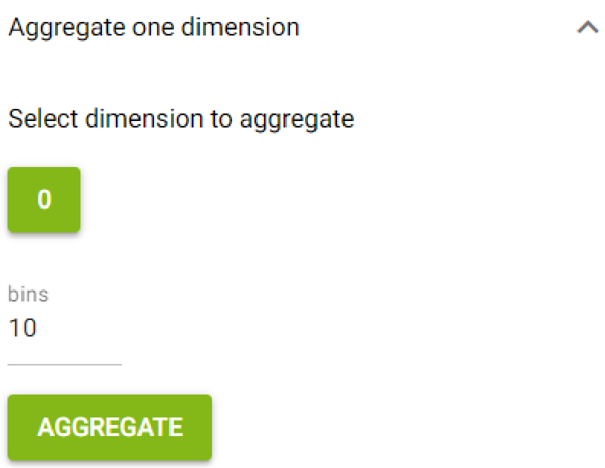


Figure 4.35: Aggregate

- `Export`: Finally, you can export the trace, as shown in Fig. 4.36. You can choose as type `yaml`, `c++` or `python` and then click `download` to export it. You can also `Download it as a compressed file`.

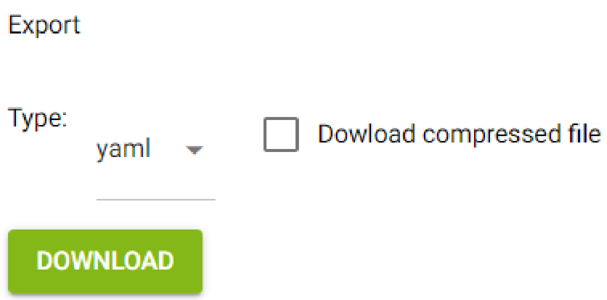


Figure 4.36: Export trace

- `FIT DISTRIBUTION OR PROCESS`: As shown in Fig. 4.37, here you can fit the distribution.

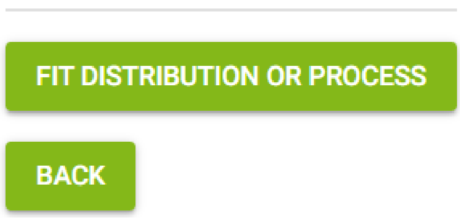


Figure 4.37: Fit distribution

After choosing `FIT DISTRIBUTION OR PROCESS`, you will be redirected to the following page as shown in Fig. 4.38.

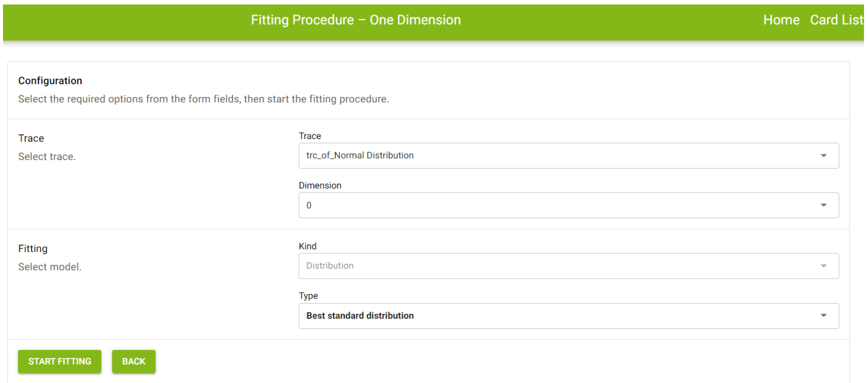


Figure 4.38: Fit distribution

At the `Trace` tab you can select `Trace`, which allows you to select the trace that you want to use for fitting and `Dimension`, which allows you to select the dimension of the trace that you want to fit.

At the `Fitting` tab, you can `Select model`, where you can select the `Kind`, which refers to what type of model or data you are fitting and `Type`, where you can specify the type of distribution to fit.

In our example, `Best standard distribution` is selected, meaning the procedure will search for the best fitted standard distribution.

Then you can choose `START FITTING` or `BACK`.

When you choose `START FITTING`, a new page will load with the fitting result. Note that this step might take some time dependent on the selected type of fitting method.

When the fitting process is finished, you will get the fitting results as shown in Fig. [4.39, 4.40].

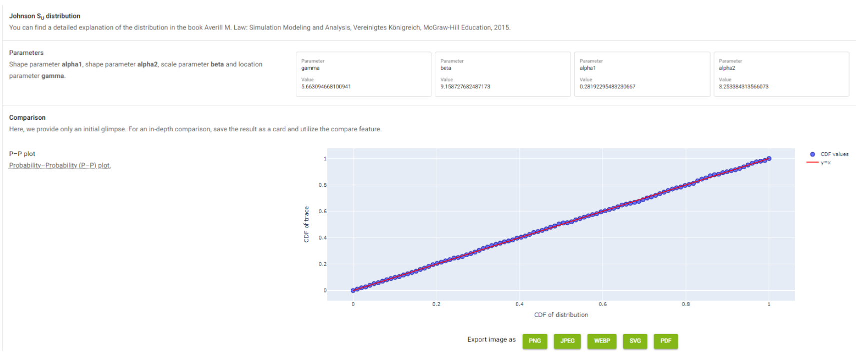


Figure 4.39: Fitting result

Figure 4.40: Fitting result

1. Distribution Fit (`Johnson SU distribution`): You can see the parameters *(alpha1, alpha2, beta, gamma)* that characterize the `Johnson SU distribution`. These parameters help shape the fitted distribution to match the trace you are analyzing.
2. `Comparison` Features:
   `P-P Plot`: This Probability-Probability plot compares the *CDF* of the trace with that of the fitted distribution.
   `Q-Q Plot`: This Quantile-Quantile plot compares the quantiles of the trace with the quantiles of the fitted distribution.
3. `Statistical tests`: Three goodness-of-fit tests (`Kolmogorov-Smirnov`, `Cramer-von Mises`, and `Chi-squared`) are provided to statistically evaluate how well the fitted distribution matches the trace. The `p-values` of these tests indicate whether the differences between the distribution and the trace are statistically significant.
4. `Log-Likelihood`: The log-likelihood value represents the likelihood of the data given the fitted distribution. A higher value indicates a better fit.

At the end of the page as shown in Fig. 4.40, you can choose what you want to do with the `fitting result`. You can choose from: `SAVE AS CARD` which allows you to save the fitted result as a card for future reference, `EXPORT AS FILE` where you can export the result as a file for further use, `GENERATE TRACE` where you can use the fitted distribution to generate a new trace based on its parameters, `GET RANDOM NUMBER GENERATOR` where you get program code for generating random numbers from the fitted distribution for simulation or modeling purposes, and finally there are the options `DISCARD` and `BACK` to leave the card.

4.2.5 Shutdown Application

This option shuts down the application, as shown in Fig. 4.41.

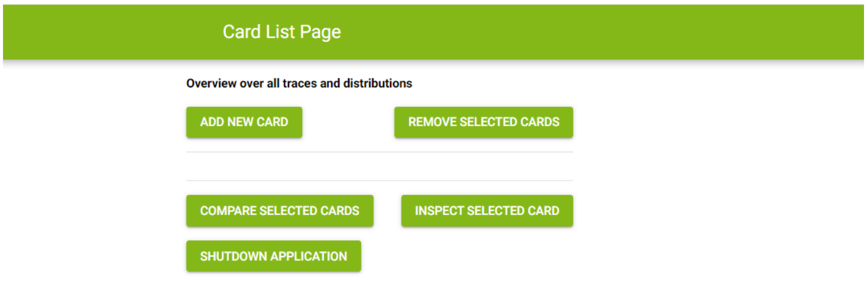


Figure 4.41: Card list

4.3 Download

At the `Download` page, as shown in Fig. 4.42, you have the options to `Download the program`, which zips the current directory and offers it for download, or `Download the trace interface`, where you can download a python module to import traces into your program and sample from the trace.

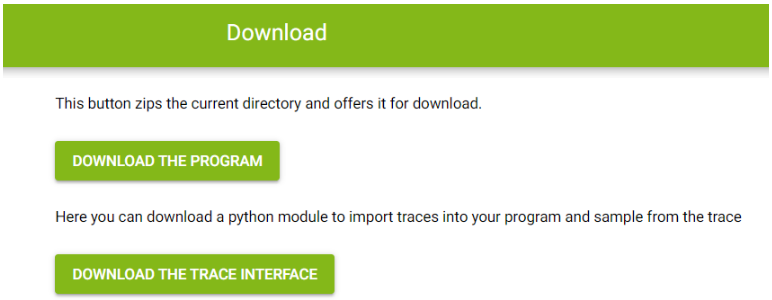


Figure 4.42: Download

A Definitions

Kurtosis

The kurtosis of a probability distribution measures the tails' heaviness relative to the normal distribution. It is given by the fourth central moment, adjusted to compare against the normal distribution.
The formula for kurtosis is:

$$g_2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^4}{n \cdot \sigma^4} - 3$$

where x_i are the observed values, $\bar{x}$ is the mean, n is the number of observations, and σ is the standard deviation. A positive value indicates heavier tails than the normal distribution, while a negative value indicates lighter tails [4].

Mean

The mean or arithmetic mean of a set of data values is a measure of central tendency. It is calculated by summing all the values and dividing by the number of observations.
The formula for the arithmetic mean is:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

where x_i are the individual data points and n is the total number of observations. The arithmetic mean is most suitable when the data is measured on a ratio or interval scale [4].

Skewness

The skewness of a frequency distribution is a measure that describes the asymmetry of the data around the mean. A distribution is said to be symmetrical if the skewness is zero. The skewness indicates the direction and the degree of asymmetry.
The formula for skewness is:

$$g_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^3}{n \cdot \sigma^3}$$

where x_i represents the data points, $\bar{x}$ is the mean, n is the number of observations, and σ is the standard deviation. A positive skewness value indicates a right-skewed distribution, while a negative value indicates a left-skewed distribution [4].

Variance

The variance is a measure of the dispersion of data points around the mean in a dataset. It quantifies how much the data varies.
The formula for variance is:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

where x_i represents individual data points, $\bar{x}$ is the mean of the data, and n is the number of observations. Variance provides insight into how spread out the data points are from the mean [4].

Distribution

A distribution refers to the function that assigns probabilities to the outcomes of a random variable. For discrete distributions, this is often done by specifying the probabilities of individual outcomes. For continuous distributions, the distribution function is often described by a probability density function (PDF).
The formula for a distribution function $F_X(x)$ is given by:

$$F_X(x) = P(X \leq x)$$

For a discrete distribution, the cumulative distribution function (CDF) is:

$$F_X(x) = \sum_{x_i \leq x} P(X = x_i)$$

For a continuous distribution, the cumulative distribution function is the integral of the probability density function $f_X(x)$:

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$$

[4].

Cumulative Distribution Function

The cumulative distribution function (CDF) $F_X(x)$ describes the probability that a random variable X will take a value less than or equal to x.
The CDF is defined as:

$$F_X(x) = P(X \leq x)$$

For a discrete random variable, the CDF is the sum of the probabilities:

$$F_X(x) = \sum_{x_i \leq x} P(X = x_i)$$

For a continuous random variable, it is the integral of the probability density function $f_X(x)$:

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt$$

The CDF is a monotonically increasing function with values ranging from 0 to 1 [4].

Probability Density Function

The probability density function (PDF) $f_X(x)$ is a function that describes the likelihood of a random variable taking on a particular value. For a continuous random variable, the PDF provides the relative likelihood of different outcomes.
The PDF is related to the cumulative distribution function (CDF) $F_X(x)$ by:

$$f_X(x) = \frac{d}{dx} F_X(x)$$

The total area under the curve of the PDF over all possible values of the random variable is equal to 1:

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

This ensures that the probabilities across all possible outcomes sum to 1 [4].

Mixture distribution

Definition (Mixture distribution [3])**.** A mixture distribution is defined by a set of $N, N \in \mathbb{N}$, component distributions specified by their probability density functions (PDFs) $f_i(x)$ and mixing probabilities $\pi_i \in [0, 1], i = 1, \ldots, N$ satisfying $\sum_{i=1}^{N} \pi_i = 1$.

Definition. The PDF of the mixture distribution is defined by

$$f(x) = \sum_{i=1}^{N} \pi_i f_i(x)$$

Phase type distribution

Definition (Phase Type Distribution [1]). A Phase Type Distribution (PHD) is defined by (π, D_0) where π is the initial distribution and D_0 is a sub-generator of an absorbing Markov chain.

Definition (Acyclic Phase Type Distribution [2]). If the transition rate matrix D_0 can be transformed into an upper (or lower) triangular matrix by symmetric permutations of rows and columns the PHD is called an Acyclic Phase Type Distribution (APHD).

Further information on PHD and APHD can be found in the literature [2].

Moments

Moments provide a more comprehensive characterization of the distribution of a random variable X than simple measures of location and spread. The k-th moment of a distribution is defined as $E[X^k]$, and the central moments as $E[(X-E[X])^k]$. The first moment is the mean $E[X]$, and the second central moment is the variance $E[(X-E[X])^2]$. From the third central moment, we derive the measure of skewness $E[(X-E[X])^3]/\sigma^3$, and from the fourth central moment, the kurtosis (or excess) $E[(X-E[X])^4]/\sigma^4 - 3$.
General formula for moments:

$$\mu_k = E[X^k]$$

[4].
The values of moments are often moderate for small values of k, but tend to be large for larger k values. Consequently, in diagrams, some form of "normalization" is often employed to avoid large values. In ASMo plots, a ratio r_k of moments is displayed, defined by

$$r_k = \frac{\mu_k}{\mu_{k-1}}$$

P-P plot

A P-P plot (Probability-Probability plot) is a graphical technique used to assess if a data set follows a given distribution. It plots the cumulative probabilities of the empirical data against the cumulative probabilities of the theoretical distribution. If the data closely follows the theoretical distribution, the points will lie approximately on the 45-degree line (the identity line).
For continuous data, the P-P plot is defined by:

$$P_P(x) = \big(F_{\text{emp}}(x_i), F_{\text{theo}}(x_i)\big)$$

where $F_{\text{emp}}(x_i)$ is the empirical cumulative distribution function (CDF), and $F_{\text{theo}}(x_i)$ is the theoretical CDF for the i-th data point [4].

Q-Q plot

A Q-Q plot is used to check if a data sample comes from a certain distribution by plotting the empirical quantiles $x_{(i)}$ against the theoretical quantiles $Q(p_i)$, where $p_i = \frac{i}{n+1}$ for $i = 1, \ldots, n$.
For a given set of observations $x_1, \ldots, x_n$, the empirical quantiles are plotted against the theoretical quantiles of a standard normal distribution. If the data is normally distributed, the points in the Q-Q plot will lie on a straight line.
The formula for the Q-Q plot is:

$$(x_{(i)}, Q(p_i)), \quad i = 1, 2, \ldots, n$$

where $x_{(i)}$ are the ordered data points and $Q(p_i)$ are the theoretical quantiles [4].

Log-likelihood

The log-likelihood function is the logarithm of the likelihood function, used in Maximum Likelihood Estimation (MLE) to estimate the parameters of a statistical model. The log-likelihood is easier to work with, particularly when maximizing the function with respect to the model's parameters.
For a set of independent and identically distributed data $x_1, x_2, \ldots, x_n$, the likelihood function $L(\theta)$ is:

$$L(\theta) = \prod_{i=1}^{n} f(x_i|\theta)$$

where $f(x_i|\theta)$ is the probability density function for each observation x_i, and θ represents the parameters to be estimated.
The log-likelihood function is:

$$\log L(\theta) = \sum_{i=1}^{n} \log f(x_i|\theta)$$

[4].

Lag k autocorrelation

The lag k autocorrelation is a measure of the relationship between observations in a time series that are separated by a lag of k time periods. It is used to assess the strength and direction of this relationship over different lags.
The formula for the empirical autocorrelation at lag k is:

$$r(k) = \frac{\sum_{t=1}^{n-k}(y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{n}(y_t - \bar{y})^2}$$

where y_t is the observation at time t, $\bar{y}$ is the mean of the time series, and n is the number of observations.
For $k = 0$, the autocorrelation is 1, meaning the observation is perfectly correlated with itself. A plot of $r(k)$ against k is called a correlogram [4].

B List of standard distributions

B.1 Continuous standard distributions

Beta distribution

ASMo

Type	Beta	
Parameters	alpha1	shape parameter
	alpha2	shape parameter

General description

Parameters	α_1	$\alpha_1 > 0$
	α_2	$\alpha_2 > 0$
PDF	$\begin{cases} \frac{x^{\alpha_1-1}(1-x)^{\alpha_2-1}}{B(\alpha_1,\alpha_2)} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$, $B(\alpha_1, \alpha_2)$ is beta function	
CDF	No closed form, in general.	
Mean	$\frac{\alpha_1}{\alpha_1 + \alpha_2}$	
Variance	$\frac{\alpha_1 \alpha_2}{(\alpha_1 + \alpha_2)^2(\alpha_1 + \alpha_2 + 1)}$	
Range	$[0, 1]$	

Erlang distribution

ASMo

Type	Erlang	
Parameters	beta	scale parameter
	k	shape parameter

General description

Parameters	β	$\beta > 0$
	k	$k > 0$
PDF	$\dfrac{x^{k-1}e^{-\frac{x}{\beta}}}{\beta^k (k-1)!}$	
CDF	$1 - \sum_{n=0}^{k-1} \dfrac{e^{-\frac{x}{\beta}} x^k}{\beta^n n!}$	
Mean	$k\beta$	
Variance	$k\beta^2$	
Range	$[0, \infty)$	

Exponential distribution

ASMo

Type	Exponential	
Parameter	beta	scale parameter

General description

Parameter	β	$\beta > 0$
PDF	$\dfrac{1}{\beta} e^{-\frac{x}{\beta}}$	
CDF	$1 - e^{-\frac{x}{\beta}}$	
Mean	β	
Variance	β^2	
Range	$[0, \infty)$	

Birnbaum–Saunders / Fatiguelife distribution

ASMo

Type	Fatiguelife	
Parameters	alpha	shape parameter
	beta	scale parameter

General description

Parameters	α	$\alpha > 0$
	β	$\beta > 0$
Mean	$\beta\left(1+\frac{\alpha^2}{2}\right)$	
Variance	$(\alpha\beta)^2\left(1+\frac{5\alpha^2}{4}\right)$	
Range	$[0, \infty)$	

Gamma distribution

ASMo

Type	Gamma	
Parameters	alpha	shape parameter
	beta	scale parameter

General description

Parameters	α	$\alpha > 0$
	β	$\beta > 0$
PDF	$\begin{cases} \frac{\beta^{-\alpha}x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$	
CDF	No closed form, in general.	
Mean	$\alpha\beta$	
Variance	$\alpha\beta^2$	
Range	$[0, \infty)$	

Gumbel distribution

ASMo

Type	Gumbel	
Parameters	beta	scale parameter
	mu	location parameter

General description

Parameters	β	$\beta > 0$
	μ	
PDF	$\frac{1}{\beta} e^{-\left(\frac{x-\mu}{\beta} + e^{-\frac{x-\mu}{\beta}}\right)}$	
CDF	$e^{-e^{-(x-\mu)/\beta}}$	
Mean	$\mu + \beta\gamma$ with γ: Euler-Mascheron constant	
Variance	$\frac{\pi^2}{6}\beta^2$	
Range	$x \in \mathbb{R}$	

Johnson S_U distribution

ASMo

Type	JohnsonSU	
Parameters	alpha1	shape parameter
	alpha2	shape parameter
	beta	scale parameter
	gamma	location parameter

General description

Parameters	α_1	
	α_2	$\alpha_2 > 0$
	β	$\beta > 0$
	γ	
PDF	$\frac{\alpha_2}{\sqrt{2\pi}\sqrt{(x-\gamma)^2+\beta^2}} e^{-\frac{1}{2}\{\alpha_1+\alpha_2 \ln\left[\frac{x-\gamma}{\beta}+\sqrt{\left(\frac{x-\gamma}{\beta}\right)^2+1}\right]\}^2}$	
CDF	$\Phi\left\{\alpha_1\alpha_2 \ln\left[\frac{x-\gamma}{\beta}+\sqrt{\left(\frac{x-\gamma}{\beta}\right)^2+1}\right]\right\}$	
Mean	$\gamma - \beta e^{1/(2\alpha_2^2)} \sinh\left(\frac{\alpha_1}{\alpha_2}\right)$	
Range	$(-\infty, \infty)$	

Log-logistic distribution

ASMo

Type	Log-logistic	
Parameters	alpha	shape parameter
	beta	scale parameter

General description

Parameters	α	$\alpha > 0$
	β	$\beta > 0$
PDF	$\begin{cases} \frac{\alpha(x/\beta)^{\alpha-1}}{\beta[1+(x/\beta)^{\alpha}]^2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$	
CDF	$\begin{cases} \frac{1}{1+(x/\beta)^{-\alpha}} & x > 0 \\ 0 & \text{otherwise} \end{cases}$	
Mean	$\beta\frac{\pi}{\alpha}\csc\left(\frac{\pi}{\alpha}\right), \alpha > 1$	
Variance	$\beta^2\frac{\pi}{\alpha}\left(2\csc\left(2\frac{\pi}{\alpha}\right) - \frac{\pi}{\alpha}\left[\csc\left(\frac{\pi}{\alpha}\right)\right]^2\right), \alpha > 2$	
Range	$[0, \infty)$	

Lognormal distribution

ASMo

Type	Lognormal	
Parameters	mu	for scale parameter e^{μ}
	sigma	shape parameter

General description

Parameters	μ	$e^{\mu} > 0$
	σ	$\sigma > 0$
PDF	$\begin{cases} \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(\ln x - \mu)^2}{2\sigma^2}\right) & x > 0 \\ 0 & \text{otherwise} \end{cases}$	
Mean	$e^{\mu+\sigma^2/2}$	
Variance	$e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$	
Range	$(0, \infty)$	

Maxwell–Boltzmann distribution

ASMo

Type	Maxwell	
Parameter	a	scale parameter

General description

Parameter	a	$a > 0, a = \sqrt{k\frac{T}{m}}$
PDF	$\sqrt{\frac{2}{\pi}} \frac{x^2}{a^3} \exp\left(\frac{-x^2}{2a^2}\right)$	
CDF	$\text{erf}\left(\frac{x}{\sqrt{2}a}\right) - \sqrt{\frac{2}{\pi}} \frac{x}{a} \exp\left(\frac{-x^2}{2a^2}\right)$ where erf is the error function	
Mean	$2a\sqrt{\frac{2}{\pi}}$	
Variance	$\frac{a^2(3\pi - 8)}{\pi}$	
Range	$(0, \infty)$	

Normal distribution

ASMo

Type	Normal	
Parameters	mu	location parameter
	sigma	scale parameter

General description

Parameters	μ	
	σ	$\sigma > 0$
PDF	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	
CDF	$\Phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$	
Mean	μ	
Variance	σ^2	
Range	$x \in \mathbb{R}$	

Pareto distribution

ASMo

Type	Pareto	
Parameters	alpha	shape parameter
	xm	scale parameter

General description

Parameters	α	$\alpha > 0$
	x_m	$x_m > 0$
PDF	$\frac{\alpha x_{\mathrm{m}}^{\alpha}}{x^{\alpha+1}}$	
CDF	$1 - \left(\frac{x_{\mathrm{m}}}{x}\right)^{\alpha}$	
Mean	$\begin{cases} \infty & \text{for } \alpha \leq 1 \\ \frac{\alpha x_{\mathrm{m}}}{\alpha-1} & \text{for } \alpha > 1 \end{cases}$	
Variance	$\begin{cases} \infty & \text{for } \alpha \leq 2 \\ \frac{x_{\mathrm{m}}^2 \alpha}{(\alpha-1)^2(\alpha-2)} & \text{for } \ alpha > 2 \end{cases}$	
Range	$[x_m, \infty)$	

Rayleigh distribution

ASMo

Type	Rayleigh	
Parameter	sigma	scale parameter

General description

Parameter	σ	$\sigma > 0$
PDF	$\frac{x}{\sigma^2} e^{-x^2/\left(2\sigma^2\right)}$	
CDF	$1 - e^{-x^2/\left(2\sigma^2\right)}$	
Mean	$\sigma\sqrt{\frac{\pi}{2}}$	
Variance	$\frac{4-\pi}{2}\sigma^2$	
Range	$[0, \infty)$	

Triangular distribution

ASMo

Type	Triangular	
Parameters	a	location parameter
	b	for scale parameter $b - a$
	m	shape parameter

General description

Parameters	a	$a < m$
	b	$b > m$
	m	$a < m < b$
PDF	$\begin{cases} \frac{2(x-a)}{(b-a)(m-a)} & \text{if } a \leq x \leq m \\ \frac{2(b-x)}{(b-a)(b-m)} & \text{if } m < x \leq b \\ 0 & \text{otherwise} \end{cases}$	
CDF	$\begin{cases} 0 & \text{if } x < a \\ \frac{(x-a)^2}{(b-a)(m-a)} & \text{if } a \leq x \leq m \\ 1 - \frac{(b-x)^2}{(b-a)(b-m)} & \text{if } m < x \leq b \\ 1 & \text{if } b < x \end{cases}$	
Mean	$\frac{a+b+m}{3}$	
Variance	$\frac{a^2+b^2+m^2-ab-am-bm}{18}$	
Range	$[a, b]$	

Uniform distribution

ASMo

Type	Uniform	
Parameters	a	location parameter
	b	for scale parameter $b - a$

General description

Parameters	a	$a < b$
	b	$b > a$
PDF	$\begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$	
CDF	$\begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } b < x \end{cases}$	
Mean	$\dfrac{a+b}{2}$	
Variance	$\dfrac{(b-a)^2}{12}$	
Range	$[a, b]$	

Weibull distribution

ASMo

Type	Weibull	
Parameters	alpha	shape parameter
	beta	scale parameter

General description

Parameters	α	$\alpha > 0$
	β	$\beta > 0$
PDF	$\begin{cases} \alpha\beta^{-\alpha}x^{\alpha-1}e^{-(x/\beta)^{\alpha}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$	
CDF	$\begin{cases} 1 - e^{-(x/\beta)^{\alpha}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$	
Mean	$\frac{\beta}{\alpha}\Gamma\left(\frac{1}{\alpha}\right)$	
Variance	$\frac{\beta^2}{\alpha}\left\{2\Gamma\left(\frac{2}{\alpha}\right) - \frac{1}{\alpha}\left[\Gamma\left(\frac{1}{\alpha}\right)\right]^2\right\}$	
Range	$[0, \infty)$	

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