Combining Qualitative and Quantitative Analysis of Generalized Stochastic Petri Nets

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Abstract. Generalized Stochastic Petri nets (GSPNs) comprise two types of transitions: timed and immediate. Since firing of immediate transitions has priority on firing of timed transitions several properties of the underlying Petri net need not be valid for the GSPN.
This article proves that liveness and the existence of home states of equal conflict nets, a generalisation of extended free choice nets, carry over to the time-augmented net iff conflicting transitions are either timed or immediate. This implies the existence of the steady state distribution of the corresponding Markov process for bounded nets.

Topics:

- Analysis and synthesis, structure and behaviour of nets
- Timed and stochastic nets

1 Introduction

Petri nets (PNs) are a formalism for modelling concurrency and synchronisation aspects inherent in modern distributed systems. Apart from modelling, they also provide a variety of techniques for analysis of qualitative (functional) properties of such systems. Most of these techniques can be applied very efficiently, e.g. invariant analysis [16] and the analysis of special net classes [5].

Since PNs do not have any notion of time, temporal specifications were incorporated emphasising those concepts which support a Markovian analysis of the time-augmented net. Two well-known representatives of this class are Stochastic Petri nets (SPNs; [15, 14]) where transitions have to fire after an exponentially distributed delay (timed transitions), and Generalized Stochastic Petri nets (GSPNs;[2]) which additionally incorporate transitions firing immediately after being enabled (immediate transitions). Here firing of immediate transitions has priority on firing of timed ones.

In SPNs all qualitative properties of the PN carry over to the time-augmented net since the reachability set and the state space of the corresponding Markov process are isomorphic. But imposing a priority relation on transitions as in GSPNs, changes the behaviour of the PN significantly, such that properties of the underlying PN need not be valid for the GSPN ([3], see also Sect. 5).

From a methodological point of view we should in general be interested in a combined qualitative and quantitative analysis of GSPNs as depicted in Fig. 1. At first, certain qualitative aspects (such as boundedness and liveness) of the Petri net would be completely analysed, neglecting the timing aspect. Then, if the Petri net satisfies all qualitative requirements, it would appear worthwhile to perform a quantitative analysis. At least two points

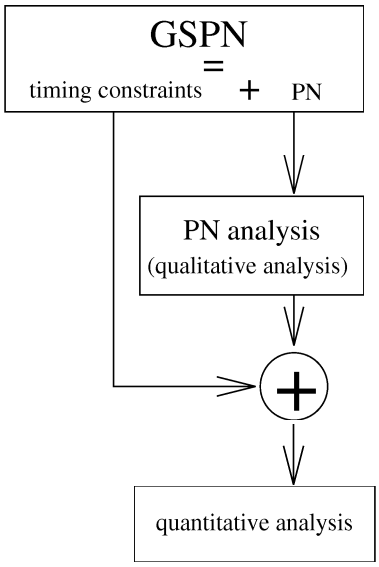

Fig. 1. Principal analysis procedure for GSPNs

motivate this procedure of analysing GSPNs: firstly the net should 'function correctly' irrespective of the timing constraints and secondly one can apply efficient Petri net algorithms directly. For this procedure to be justifiable, all qualitative features of the Petri net would have to remain valid after the introduction of time.

As mentioned before this is not the case. So the analysis procedure in Fig. 1 has to be modified or possibly completely changed to yield a useful combination of qualitative and quantitative analysis. What possibilities do we have to cope with this basic problem?

One way is to modify existing Petri net algorithms to render them suitable for analysis of the GSPN; this might turn out a rather uncomfortable approach, as Petri net theory could no longer be used directly. Furthermore, all future research in the Petri net area would have to be adapted to GSPNs.

Another way is to develop suitable restrictions for an integration of timing aspects, such that qualitative properties remain valid. This idea leads to the analysis procedure given in Fig. 2. The great benefit of this is that the standard theory of Petri nets remains unaffected: additionally, any forthcoming results on efficient algorithms for qualitative analyses would be applicable without difficulty.

This article shows that in GSPNs exhibiting an equal conflict net structure [18], several properties of the underlying Petri net remain valid for the GSPN iff conflicting transitions are of the same kind, either timed or immediate. In this article we extend the theorems presented in [3] and furthermore greatly simplify a proof there showing the existence of home states.

The outline of the article is as follows. Section 2 recalls some basic definitions of Petri nets and the definition of GSPNs is given in Sect. 3. Section 4 describes the quantitative analysis of GSPNs and shows that liveness and the existence of home states are essential preconditions. Section 5 presents several examples illustrating discrepancies between qualitative properties of the GSPN and its underlying PN. Section 6 establishes the announced results.

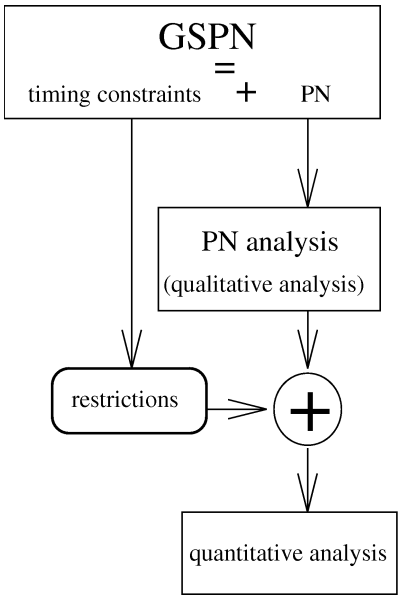

Fig. 2. Modified analysis procedure for GSPNs

2 Basic definitions

We will use the notion 'Petri nets' (PNs) for Place/Transition-nets (P/T-nets) [16].

Definition 1 Place/Transition-net. A *Place/Transition-net* is a 4-tuple $PN = (P, T, I^-, I^+)$ where

- P is a finite and non-empty set of places,
- T is a finite and non-empty set of transitions,
- $P \cap T = \emptyset$,
- $I^-, I^+ : P \times T \to \mathbb{N}_0$ are the backward and forward incidence functions.

We will use the pair (PN, M_0) to denote the marked net with initial marking M_0.

Definition 2 Basic notions of P/T-nets.
Let $PN = (P, T, I^-, I^+)$ be a P/T-net.

1. $t\bullet := \{p \in P \mid I^+(p,t) > 0\}$, $\bullet t := \{p \in P \mid I^-(p,t) > 0\}$,
   $p\bullet := \{t \in T \mid I^-(p,t) > 0\}$, $\bullet p := \{t \in T \mid I^+(p,t) > 0\}$
   and the usual extension to sets $X \subseteq P \cup T$ is defined as
   $\bullet X := \bigcup_{x \in X} \bullet x, X\bullet := \bigcup_{x \in X} x\bullet$.
2. Two transitions $t, t' \in T$ are in (*structural*) *conflict* iff $\bullet t \cap \bullet t' \neq \emptyset$.
3. A *marking* of a P/T-net is a function $M : P \mapsto \mathbb{N}_0$, where $M(p)$ denotes the number of tokens in p.
4. A set $\tilde{P} \subseteq P$ is *marked* in a marking M, iff $\exists p \in \tilde{P} : M(p) > 0$; otherwise $\tilde{P}$ is *unmarked* or *empty* in M.
5. A transition $t \in T$ is *enabled* at M, denoted by $M[t>$, iff $M(p) \geq I^-(p,t), \forall p \in P$.
6. A transition $t \in T$ being enabled at marking M *may fire* yielding a new marking given by $M'(p) = M(p) - I^-(p,t) + I^+(p,t), \forall p \in P$, denoted by $M[t > M'$. M' is *directly reachable* from M denoted by $M \to M'$. Let $\to^*$ be the reflexive and transitive closure of $\to$. A marking M' is *reachable* from M, iff $M \to^* M'$.

7. The *reachability set* of (PN, M_0) is defined by $R(PN, M_0) := \{M \mid M_0 \rightarrow^* M\}$.
8. $\tilde{R} \subseteq R(PN, M_0)$ is a *final strongly connected component* of $R(PN, M_0)$ iff $\forall M \in \tilde{R}, M' \in R(PN, M_0) : M' \in R(PN, M) \Longrightarrow (M \in R(PN, M')$ and $M' \in \tilde{R})$.
9. A *firing sequence* of PN is a finite sequence of transitions $f = t_1 \ldots t_n, n \geq 0$ such that there are markings $M_1, \ldots, M_{n+1}$ satisfying $M_i[t_i > M_{i+1}, \forall i = 1, \ldots, n$. A shorthand notation for this case is $M_1[f >$ and $M_1[f > M_{n+1}$ respectively. f has *concession* at M iff $M[f >$ holds. ε denotes the empty firing sequence and $M[\varepsilon > M$ holds for all markings M. If a transition t is part of a firing sequence f this will be denoted by $t \in f$.
10. (PN, M_0) is *bounded*, iff $\forall p \in P : \exists k \in \mathbb{N}_0 : \forall M \in R(PN, M_0) : M(p) \leq k$.
11. (PN, M_0) is *live*, iff $\forall t \in T, M \in R(PN, M_0) : \exists M' \in R(PN, M) : M'[t >$.
12. A marking $M \in R(PN, M_0)$ is a *home state*, iff $\forall M' \in R(PN, M_0) : M \in R(PN, M')$.
13. PN is a *free choice net (FC-net)*, iff $\forall p \in P, t \in T : I^-(p,t) \leq 1, I^+(p,t) \leq 1$ and $\forall t, t' \in T : {}^\bullet t \cap {}^\bullet t' = \emptyset$ or $|{}^\bullet t| = |{}^\bullet t'| = 1$.
14. PN is an *extended free choice net (EFC-net)*, iff $\forall p \in P, t \in T : I^-(p,t) \leq 1, I^+(p,t) \leq 1$ and $\forall t, t' \in T : {}^\bullet t \cap {}^\bullet t' = \emptyset$ or ${}^\bullet t = {}^\bullet t'$.
15. PN is an *equal conflict net (EC-net)*, iff $\forall t, t' \in T : {}^\bullet t \cap {}^\bullet t' = \emptyset$ or $I^-(p,t) = I^-(p,t'), \forall p \in P$.

EC-nets are a generalisation of EFC-nets and they unveil similar properties. [18], e.g., proves the existence of home states in live and bounded EC-nets. EC-nets exhibit the same characteristic as EFC-nets, namely that if a transition is enabled then all conflicting transitions are enabled as well. We will use this property extensively in Sect. 6.

3 Generalized Stochastic Petri Nets

Generalized Stochastic Petri nets [1, 2] are obtained by allowing transitions to belong to two different classes: immediate transitions and timed transitions. Immediate transitions fire in zero time once they are enabled. Timed transitions fire after a random, exponentially distributed enabling time. Timed transitions are drawn as boxes and immediate transitions are drawn as bars. Novel definitions of GSPNs (cf. [9]) also incorporate inhibitor arcs and different priority levels for immediate transitions. PNs with inhibitor arcs have the power of Turing machines [16] and problems like reachability and liveness problem become undecidable. In bounded PNs a net with inhibitor arcs can always be represented by a semantically equivalent P/T-net. Thus in the definition given here inhibitor arcs and for simplicity several priority levels are excluded adopting the GSPN formalism in [2].

Definition 3. A *GSPN* (cf. [1, 2]) is a 4-tuple
$GSPN = (PN, T_1, T_2, W)$ where

- $PN = (P, T, I^-, I^+)$ is the underlying P/T-net
- $T_1 \subseteq T$ is the set of timed transitions, $T_1 \neq \emptyset$,
- $T_2 \subset T$ denotes the set of immediate transitions, $T_1 \cap T_2 = \emptyset$, $T = T_1 \cup T_2$
- $W = (w_1, \ldots, w_{|T|})$ is an array whose entry w_i
  - is a (possibly marking dependent) rate $\in \mathbb{R}^+$ of a negative exponential distribution specifying the firing delay, if transition t_i is a timed transition, i.e. $t_i \in T_1$ or
  - is a (possibly marking dependent) weight $\in \mathbb{R}^+$ specifying the relative firing frequency, if transition t_i is an immediate transition, i.e. $t_i \in T_2$.

Similar to the underlying Petri net we will use the pair $(GSPN, M_0)$ to specify the marked net with initial marking M_0. If obvious from the context, the marked net will also be denoted by $GSPN$.
According to the definition of GSPNs the enabling of transitions is defined as follows.

Definition 4. A transition $t \in T$ of a GSPN is *GSPN-enabled* at M, denoted by $M[t >_{GSPN}$, iff $M(p) \geq I^-(p,t), \forall p \in P$ and $(\exists t' \in T_2 : M(p) \geq I^-(p,t'), \forall p \in P \Longrightarrow t \in T_2)$.

I.e. a transition is enabled iff the usual enabling rule for P/T-nets holds and if there exists an immediate transition t' being enabled in the sense of P/T-nets, then t must also be an immediate transition, since otherwise the firing of t' has priority on the firing of t.

With respect to the enabling rule in GSPNs, all definitions given for P/T-nets (cf. Def. 2) can be defined accordingly for GSPNs. To distinguish between the corresponding notions for the underlying Petri net, we will use subscripts PN and $GSPN$, e.g. $M[t >_{PN}, M[t >_{GSPN}, M[f >_{PN}$, $M \rightarrow^*_{GSPN} M'$ etc. or $R(GSPN, M_0)$ etc.

Several transitions may be simultaneously enabled at a marking M. The set of enabled transitions, denoted by $EN_T(M)$, is defined as follows.

Definition 5. Let M be a marking of the GSPN. $EN_T(M) := \{t \in T | M[t >_{GSPN}\}$

4 Quantitative analysis of GSPNs

This section briefly summarises the quantitative analysis of GSPNs (cf. [2]). The state space of a GSPN is the disjunct union of tangible and vanishing states, denoted by $\hat{T}$ and $\hat{V}$ respectively. In tangible (vanishing) states only timed (immediate) transitions are enabled. So $R(GSPN, M_0) = \hat{T} \cup \hat{V}$ where $\hat{T} = \{M \in R(GSPN, M_0) | EN_T(M) \cap T_2 = \emptyset\}$ and $\hat{V} = R(GSPN, M_0) \setminus \hat{T}$. In the following the notions 'state' and 'marking' are used synonymously and we assume that the state space is finite. The probability p_{ij} of changing from $M_i \in R(GSPN, M_0)$ to $M_j \in R(GSPN, M_0)$, if $M_i[t_k >_{GSPN} M_j$ for some $t_k \in T$ is given by

$$p_{ij} = \frac{w_k}{\sum_{n:t_n \in EN_T(M_i)} w_n}$$

Corresponding to the separation of the state space in two subsets we can define four matrices $C = (p_{ij})_{M_i \in \hat{V}, M_j \in \hat{V}}$, $D = (p_{ij})_{M_i \in \hat{V}, M_j \in \hat{T}}$, $E = (p_{ij})_{M_i \in \hat{T}, M_j \in \hat{V}}$ and $F = (p_{ij})_{M_i \in \hat{T}, M_j \in \hat{T}}$.

Since the sojourn time in vanishing states is zero by definition, we are only interested in tangible states. Thus it is sufficient to solve the global balance equations of the reduced embedded Markov chain given by

$$\tilde{\pi} = \tilde{\pi}(F + E\sum_{k=0}^{\infty} C^k D); \quad \sum_{j:M_j \in \hat{T}} \tilde{\pi}_j = 1, \tag{1}$$

provided $\sum_{k=0}^{\infty} C^k = (I - C)^{-1}$ exists. If (1) has a unique solution the steady state distribution of the GSPN's stochastic process is given by

$$\pi_j = \begin{cases} 0 & \text{if } M_j \in \hat{V} \\ \dfrac{\tilde{\pi}_j \left(\sum\limits_{k\,:\,t_k \in EN_T(M_j)} w_k \right)^{-1}}{\sum\limits_{n\,:\,M_n \in \hat{T}} \tilde{\pi}_n \left(\sum\limits_{k\,:\,t_k \in EN_T(M_n)} w_k \right)^{-1}} & \text{if } M_j \in \hat{T} \end{cases} \tag{2}$$

and performance measures can be computed from this steady state distribution as usual.

The description above shows that two important preconditions have to be satisfied for a successful quantitative analysis of GSPNs:

a) $\sum_{k=0}^{\infty} C^k$ must exist and
b) Equation (1) must have a unique solution.

Both requirements can be expressed in terms of qualitative properties of the GSPN.

The existence of $\sum_{k=0}^{\infty} C^k$ corresponds to the notion 'trap'. Define $u_i := 1 - \sum_{j=1}^{|\hat{V}|} c_{ij}$. $u_i > 0$ if the sum in row i of matrix C is less then 1 and $u_i = 0$ if it is 1. We can separate the set of vanishing states into two subsets $J_0 := \{M_i | u_i = 0\}$ and $J_1 := \{M_i | u_i > 0\}$. If the stochastic process is in a state $M_i \in J_0$, it can only transit to another vanishing state, being in a state $M_j \in J_1$ there is a positive probability to reach a tangible state. The existence of the limit in (1) can be analysed by inspecting the structure of the reachability set with respect to vanishing states. If the process can always reach tangible states, then this limit exists.

Definition 6 trap; cf. [17].
C has no *trap* $:\Longleftrightarrow \forall M_i \in J_0 : \exists M_j \in J_1 : M_j \in R(GSPN, M_i)$.
Otherwise C has a trap.

Theorem 7 [17]. *C has no trap* $\Longleftrightarrow (I - C)^{-1} = \sum_{k=0}^{\infty} C^k$ *exists.*

Since C consists of the transition probabilities between vanishing states, we speak of a timeless trap. If there is no timeless trap, we can determine the global balance equations of the reduced embedded Markov chain. Note that a timeless trap can only exist if the GSPN is not live, since all timed transitions are not live. According to the firing rule in GSPNs a timeless trap can be defined as follows:

Definition 8. A GSPN has a *timeless trap*, iff $\exists M \in R(GSPN, M_0)$:

1. $\forall k \in \mathbb{N} : \exists g \in T_2^* : |g| \geq k$ and $M[g >_{GSPN}$
2. $\forall f \in T^* : (M[f >_{GSPN} \Longrightarrow f \in T_2^*)$

where $|g|$ denotes the length of g. Since T is a finite set, a timeless trap implies that some immediate transitions fire infinitely often. Figure 3 depicts an example of a timeless trap, which is constituted by firings of the immediate transitions t_7 and t_8. Note that the definition of a timeless trap directly coincides with the notion of a trap of matrix C.

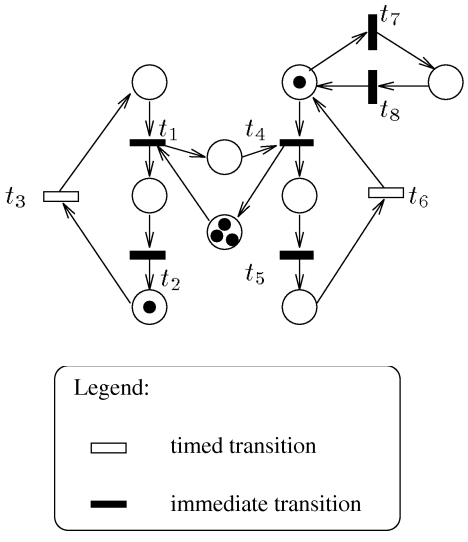

Fig. 3. GSPN with a timeless trap

Another condition for the existence of a steady state distribution is the existence of a unique solution of (1), i.e. the finite reachability set of the GSPN has to contain exactly one final strongly connected component [10]. Such a component exists iff the GSPN has at least one home state.

5 Qualitative Analysis of GSPNs

The former section demonstrated, that the absence of timeless traps, liveness and the existence of home states, are essential preconditions for the existence of a steady state distribution. In the qualitative analysis of a GSPN we obviously want to exploit the underlying Petri net part of the GSPN as discussed in the introduction. Unfortunately the incorporation of timing aspects, especially introducing a priority relation on transitions, changes the properties of the Petri net significantly. In this section we will have a closer look on this phenomenon.

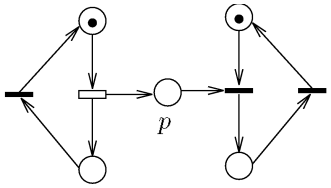

Fig. 4. GSPN bounded, PN not bounded

An unbounded Petri net, e.g., may yield a bounded GSPN. The PN of Fig. 4 contains an unbounded place p, but the GSPN is bounded. Since $R(GSPN, M_0) \subseteq R(PN, M_0)$ the reverse, fortunately, does not present a similar problem. According to the analysis procedure proposed in Fig. 1 this is a desired result, since a "positive" qualitative property of the PN carries over to the GSPN irrespective of the timing constraints imposed on the net.

Other properties unveil greater problems. E.g. there is no implication between the liveness of a GSPN and its underlying Petri net. This fact is well-known for deterministically timed nets [12]. Figure 5 shows a non-live GSPN, whose PN is live. t is only enabled at the initial marking which is the only marking where p_1 and p_2 are marked simultaneously. Note that the initial marking is not a home state. Figure 6 on the other hand depicts a live GSPN with a non-live underlying Petri net. After firing t_2 and t_3 the PN is dead. With regard to the GSPN the firing sequence $t_2 t_3$ can not occur since t_1 is an immediate transition having priority on the firing of t_2. According to the analysis procedure depicted in Fig. 1 we are especially interested in conclusions concerning the liveness of a GSPN whose underlying PN is live. The following lemma shows that total deadlocks can not occur in such GSPNs.

Lemma 9. *Let GSPN be a Generalized Stochastic Petri net, whose underlying Petri net is live. Then* $\forall M \in R(GSPN, M_0) : \exists t \in T : M[t >_{GSPN}$.

Proof. Since the enabling rule of GSPNs (cf. Def. 4) is a restriction of the enabling in P/T-nets, we get $R(GSPN, M_0) \subseteq R(PN, M_0)$. Liveness of the underlying Petri net implies: $\forall M \in R(GSPN, M_0) : \exists t \in T : M[t >_{PN}$. If $t \in T_2$ then also $M[t >_{GSPN}$ holds and we are done. If $t \in T_1$ then $\neg M[t >_{GSPN}$ might only hold if there is an immediate transition, which prevents the enabling of t, i.e. $\exists t' \in T_2 : M[t' >_{GSPN}$ and thus t' is GSPN-enabled at M. □

So in such GSPNs there is always some enabled transition, although there might be no live transition (see Fig. 7), i.e. $\nexists t \in T : \forall M \in R(GSPN, M_0) : \exists M' \in R(GSPN, M)$ and $M'[t >_{GSPN}$.

Furthermore, although the PN contains home states, this may not hold for the GSPN. Figure 8 illustrates such a situation. After firing transition t_1 or t_2 the GSPN 'enters' one of two final strongly connected components of the reachability set. These are characterised by markings of the form $(2, 0, \ldots)$ and $(0, 2, \ldots)$ respectively which are part of only one component.

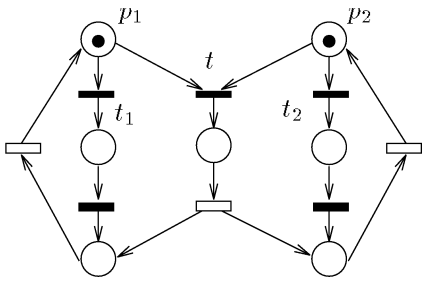

Fig. 5. PN live, GSPN not live

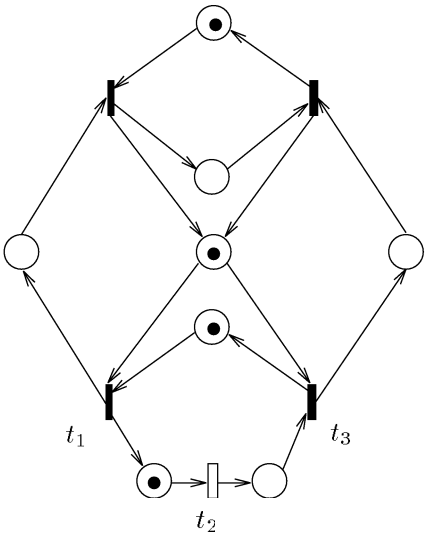

Fig. 6. GSPN live, PN not live

All these examples illustrate that the analysis procedure in Fig. 1 has to be modified. As described in the introduction we head for a restriction on the possible timing constraints such that a combined qualitative and quantitative analysis of GSPNs as depicted in Fig. 2 becomes practicable. Finding such restrictions for general net structures is difficult. We restrict ourselves to EC-nets. Another reason for this limitation is that a subclass of EC-nets, namely EFC-nets, has been studied exhaustively [5, 7] and qualitative properties can be characterised by efficiently testable conditions, e.g. [13].

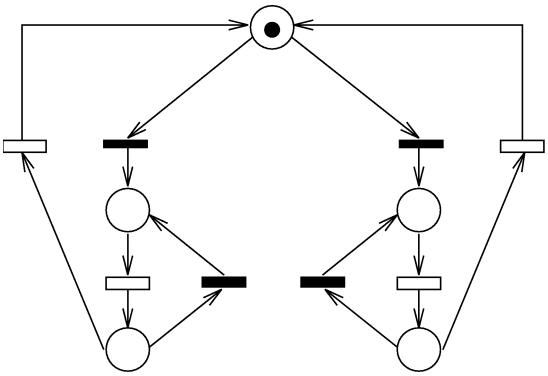

Fig. 7. GSPN with no live transitions

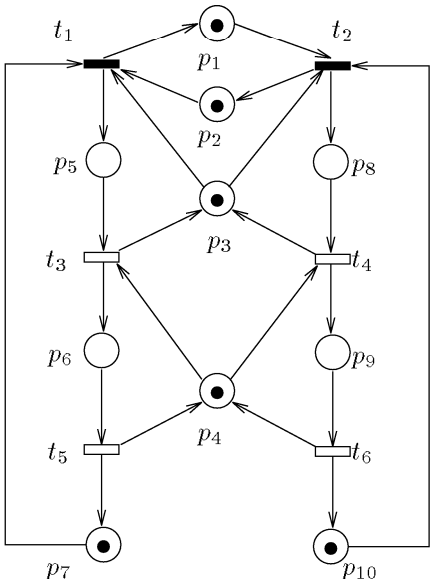

Fig. 8. GSPN with no home states

6 Qualitative Analysis of EC-GSPNs

Definition 10. A GSPN is an *EC-GSPN*, iff its underlying Petri net is an EC-net.

Since in EC-nets conflicting transitions are enabled simultaneously, it seems obvious to demand that conflicting transitions should be either timed or immediate. In fact we need to impose only one condition on an EC-GSPN, which we shall call EQUAL-Conflict, such that the analysis procedure of Fig. 2 is applicable.

Definition 11. A $GSPN = (PN, T_1, T_2, W)$ satisfies condition *EQUAL-Conflict*, iff $\forall t, t' \in T : {\bullet t} \cap {\bullet t'} \neq \emptyset \Longrightarrow \{t, t'\} \subseteq T_1$ or $\{t, t'\} \subseteq T_2$.

Condition EQUAL-Conflict exactly states that conflicts may only occur between transitions of the same kind. Obviously this condition is important in the analysis of the liveness of a GSPN. We will show that it ensures the absence of timeless traps and that liveness and the existence of home states carry over to the GSPN.

Theorem 12. *If we are given an EC-GSPN whose underlying Petri net is live and bounded, then: Condition EQUAL-Conflict $\Longrightarrow$ GSPN has no timeless trap.*

Proof. Assume the GSPN has a timeless trap, because of lemma 9 i.e.
$\exists M \in R(GSPN, M_0) : \forall f \in T^* : (M[f >_{GSPN} \Longrightarrow f \in T_2^*)$.
Since PN is bounded the GSPN is bounded as well. Thus $R(GSPN, M_0)$ is finite and there exists a final strongly connected component (cf. Def. 2) $C_M \subseteq R(GSPN, M_0)$.
Define $\tilde{T} := \{t \in T | \exists M' \in C_M : M'[t >_{GSPN}\}$ and $\tilde{P} := {}^\bullet\tilde{T}$.

$\tilde{T}$ can be referred to as the set of live transitions with regard to C_M. Because of our assumption we have $\tilde{T} \neq \emptyset$ and $\tilde{T} \subseteq T_2$. Since the Petri net is live and bounded, it is strongly connected [5]. So one of the following two cases holds (see Fig. 9):

a) $\exists p_r \in \tilde{P} : p_r{}^\bullet \supseteq \{t_i, t_j\}$ and $t_i \in \tilde{T}, t_j \in T \setminus \tilde{T}$.
The assumption of a timeless trap yields $t_i \in T_2$ and thus if $M'[t_i >_{GSPN}$ for some marking $M' \in C_M$, the EC-net structure and condition EQUAL-Conflict imply $M'[t_j >_{GSPN}$ contradicting $t_j \notin \tilde{T}$.

b) $\exists t_r \in \tilde{T} : t_r{}^\bullet \supseteq \{p_k, p_l\}$ and $p_k \in \tilde{P}, p_l \in P \setminus \tilde{P}$.
Since $t_r \in \tilde{T}$, t_r fires infinitely often with regard to C_M and our assumption of a timeless trap. Because of $p_l \notin \tilde{P} = {}^\bullet\tilde{T}$, place p_l is not bounded, contradicting the boundedness of the GSPN.

Thus our assumption is not valid, which completes the proof. □

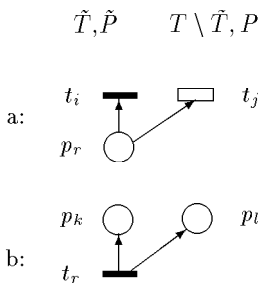

Fig. 9. Illustration of proof

In the following we will prove that condition EQUAL-Conflict is sufficient for ensuring the existence of home states for an EC-GSPN whose underlying Petri net is live and bounded. The proof of this theorem requires some preparations. During the proof we will additionally see that condition EQUAL-Conflict is sufficient and necessary for liveness.

We use the fact that the existence of home states is equivalent to the directedness property [8], which is:

$$R(GSPN, M) \cap R(GSPN, M') \neq \emptyset, \forall M, M' \in R(GSPN, M_0)$$

First of all let us consider an example to get an impression how the firing of transitions is influenced by the two kinds of transitions appearing in GSPNs. For simplifying notation markings are denoted by formal sums over the set of places. Figure 10 depicts a live and bounded EC-GSPN with initial marking $M_0 = p_1$. Consider two markings $M = p_2 + p_3$ and $M' = p_3 + p_4$ of the GSPN's reachability set ($M_0[t_1 >_{GSPN} M$ and $M_0[t_2 >_{GSPN} M'$). Since the underlying Petri net is a live and bounded EC-net it has home states [18] and thus the directedness property holds. E.g. $M'' = p_3 + p_8 \in R(PN, M) \cap R(PN, M')$ is a

marking reachable from M and M' after occurrence of the firing sequences $f = t_3$ and $g = t_4t_5$ (see Fig. 11 and Fig. 12 for the general case). Both firing sequences do not have concession in the GSPN, since transition t_6 is GSPN-enabled at M and at M'. Concerning firing sequence g the firing of immediate transitions ($h = t_6t_7$) interleaves with the firing of transitions in g. We can construct a firing sequence $\tilde{g}$ comprising all transitions of g and additionally some immediate transitions such that $M'[\tilde{g} >_{GSPN}$ and $M''[h >_{GSPN}$ yielding a marking $\tilde{M'} = p_6 + p_7 + p_8 \in R(GSPN, M') \cap R(GSPN, M'')$. $\tilde{g}$ can be constructed stepwise as follows. At M', t_6 is the only GSPN-enabled transition. Afterwards t_5, t_7 and t_9 are GSPN-enabled. Since t_5 is part of g it is obvious to select this transition next and its firing yields a marking $p_4 + p_5 + p_8$ where t_7 and t_9 are both GSPN-enabled. Choosing t_7 leads to a marking where finally t_4 is GSPN-enabled. Note that the sequence of transitions of g is permuted in $\tilde{g}$.

Similarly we can find a firing sequence $\tilde{f} = t_6t_5t_7t_3$ which is a permutation of f,h and $\tilde{h} = t_5$. $\tilde{h}$ is GSPN-enabled at $\tilde{M'}$. This leads to a marking $\tilde{M''} = p_7 + 2p_8 \in R(GSPN, M) \cap R(GSPN, M')$ illustrating that the directedness property holds for M and M' with reference to the GSPN.

During construction of the firing sequences h and $\tilde{h}$ we have to carefully select the corresponding immediate transitions. Consider, e.g., the marking $p_4 + p_5 + p_8$ we have encountered during construction of $\tilde{g}$. In that marking we have the choice between t_7 and t_9 and selection of t_9 and t_{10} might have resulted in a non-terminating construction. We can avoid such situations if transitions (not part of g) have to be selected from a circuit-free allocation on the subset of immediate transitions. Such an allocation will be called a T_1-allocation.

The example implies the following idea for proving the existence of home states (cf. Fig. 12). Since live and bounded EC-nets have home states we have the following directedness property: $\forall M, M' \in R(GSPN, M_0) : \exists M'' \in R(PN, M) \cap R(PN, M')$. Choose two arbitrary markings $M, M' \in R(GSPN, M_0)$. We have to show that the directedness property holds for the reachability graph of the GSPN as well, i.e. $\exists \tilde{M''} \in R(GSPN, M) \cap R(GSPN, M')$. Let $f, g \in T^*$ be two firing sequences with $M[f >_{PN} M''$ and $M'[g >_{PN} M''$. We will then construct firing sequences $h \in T_2^*$ and $\tilde{g} \in Perm(gh)$ such that $M''[h >_{GSPN}$ and $M'[\tilde{g} >_{GSPN}$ where $Perm(g)$ denotes the set of all firing sequences which are permutations of g. Similarly we construct firing sequences $\tilde{h} \in T_2^*$ and $\tilde{f} \in Perm(fh\tilde{h})$ with $\tilde{M'}[\tilde{h} >_{GSPN}$ and $M[\tilde{f} >_{GSPN}$ yielding the existence of a marking $\tilde{M''} \in R(GSPN, M) \cap R(GSPN, M')$. For the proof we need the following definitions:

Definition 13.

a) difference of two firing sequences (cf. [17]). $f, g \in T^*, t \in T$.

$$f - \varepsilon := f$$

$$f - t := \begin{cases} f & \text{if } t \notin f \\ f_1 f_2 & \text{if } f_1, f_2 \in T^* : t \notin f_1 \text{ and } f = f_1 t f_2 \end{cases}$$

$$f - tg := (f - t) - g$$

b) permutation. The set $Perm(f)$ of all permutations of $f \in T^*$ is
$Perm(f) := \{g \in T^* \mid (f - g)(g - f) = \varepsilon\}$.

c) allocation. An *allocation* α is a mapping $\alpha : P \mapsto T$ with $\alpha(p) \in p\bullet, \forall p \in P$.

d) T_1-allocation. An allocation α is a *T_1-allocation* iff $\forall p_0 \in P : \alpha(p_0) \in T_1$ or
$\exists p_1, \ldots, p_k \in P :$ with $p_{i+1} \in \alpha(p_i)\bullet, \forall i = 0, \ldots, k-1$ and $\alpha(p_k) \in T_1$.

Subtracting g from f cancels all transitions of g in f, provided they are part of f, where cancelling starts at the "beginning" of f. E.g. $t_2t_1t_1t_3t_5t_4 - t_4t_7t_1t_3 = t_2t_1t_5$. As usual concatenation has precedence on difference, i.e. $fg - h = (fg) - h$.
Firing transitions of a T_1-allocation directs the token flow to timed transitions. Obviously T_1-allocations exist for every strongly connected net.

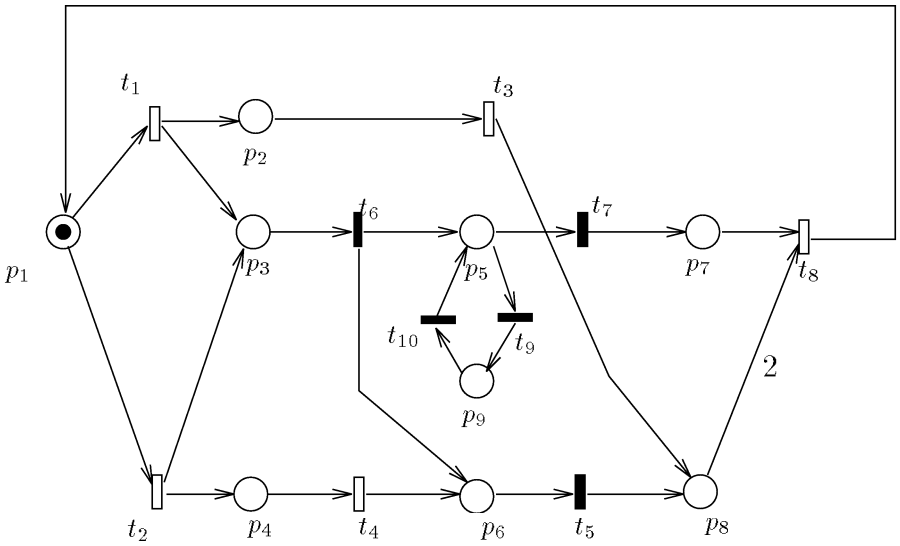

Fig. 10. Example of an EC-GSPN

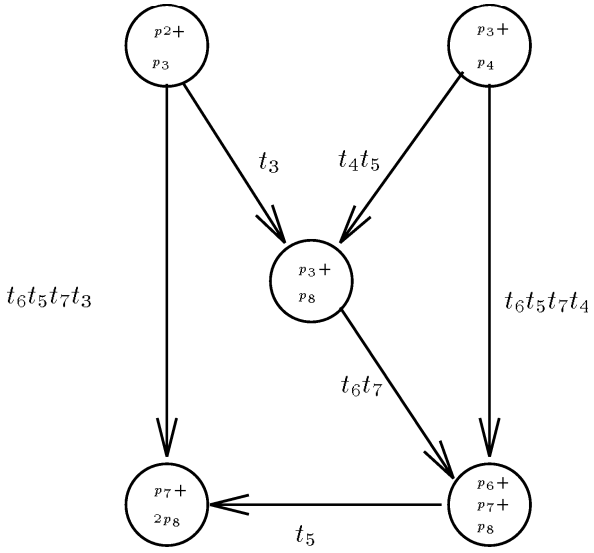

Fig. 11. Part of the reachability graph

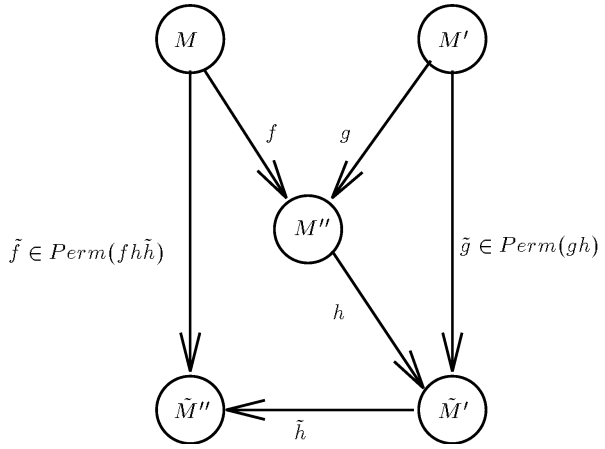

Fig. 12. Existence of home states (idea of proof)

Lemma 14. *If GSPN is strongly connected, then there exists a T_1-allocation α.*

Proof. We can construct such an allocation as follows.
Define $\alpha(p) = t, \forall t \in T_1, p \in \bullet t$. If a place has more than one timed output transition choose an arbitrary timed transition.
If α is defined for $p \in P$, but not defined for some $p' \in \bullet(\bullet p)$, define $\alpha(p') = t$ for some arbitrary $t \in p' \bullet \cap \bullet p$.
Since P is finite, this construction procedure terminates and obviously α is a T_1-allocation. □

Lemma 15. *Let GSPN be bounded, α an allocation and $M \in R(GSPN, M_0)$ some marking.*
If there is an infinite sequence of transitions $(t_{i_j})_{j \in \mathbb{N}_0}$ with $t_{i_j} \in \alpha(P) \cap T_2$ and $\forall j \in \mathbb{N}$: $M[t_{i_1} \ldots, t_{i_j} >_{GSPN}$
then α is not an T_1-allocation.

Proof. Since T is finite, there exists at least one transition $t \in (t_{i_j})_{j \in \mathbb{N}_0}$ which fires infinitely often, i.e.

$$\exists t \in T : \forall k \in \mathbb{N} : \exists k' \in \mathbb{N}, k' > k : t = t_{i_{k'}}$$

Choose an arbitrary place $p \in t\bullet$. Since α is an T_1-allocation there exists $\tilde{t} \in T_1$ with

a) $\alpha(p) = \tilde{t}$ or
b) $\exists p_0, \ldots, p_k \in P, t_0, \ldots, t_k \in T : p_0 = p, t_k = \tilde{t}, t_i \in \bullet p_{i+1}, \forall i = 0, \ldots, k-1$ and $\alpha(p_i) = t_i, \forall i = 0, \ldots, k$.

In case of a) p is not bounded, since t fires infinitely often. In case of b) we can find transitions t_i, t_{i+1} where t_i fires infinitely often and t_{i+1} occurs only a finite number of times. This holds, because t fires infinitely often and $\tilde{t}$ is excluded from firing, since $\tilde{t} \in T_1$. Similar to the first case there exists a place $p \in t_i \bullet \cap \bullet t_{i+1}$ being not bounded. □

Lemma 15 asserts that the construction process terminates if we select all additionally firing transitions from a T_1-allocation. The next lemma states a well-known property of non-conflicting transitions concerning a reordering of firing sequences.

Lemma 16. *Let PN be a P/T-net and M denote some marking.*

a) *$t, t' \in T : \bullet t \cap \bullet t' = \emptyset$. Then $M[t >_{PN}, M[t' >_{PN} \Longrightarrow M[tt' >_{PN}$.*
b) *$t \in T, h \in T^* : \forall t' \in h : \bullet t \cap \bullet t' = \emptyset$. Then $M[ht >_{PN}, M[t >_{PN} \Longrightarrow M[th >_{PN}$.*
c) *Given $f, g, q \in T^*, h \in Perm(f - g)$ where $\forall t \in g, t' \in q : \bullet t \cap \bullet t' = \emptyset$ we have $M[f >_{PN}, M[hq >_{PN} \Longrightarrow M[fq >_{PN}$.*

Proof. **a)+b)** [19], Lemma 3.1.
c) Obvious, since all transitions of g are not in conflict with transitions of q. So additional firing of transitions in g does not disable transitions of q. □

The next lemma is the key for proving liveness and the existence of home states in EC-GSPNs. It states that if a firing sequence has concession in the Petri net then a permutation of this firing sequence with additionally interleaved immediate transitions has concession in the EC-GSPN provided condition EQUAL-Conflict holds.

Lemma 17. *Let GSPN be an EC-GSPN satisfying condition EQUAL-Conflict and whose underlying Petri net is live and bounded. Let $M \in R(GSPN, M_0)$ be a marking of the GSPN, α a T_1-allocation and $f = t_1 \ldots t_k \in T^*$ such that $M[f >_{PN} M'$ for some marking M'. Then the following holds:*
$\exists \tilde{f} = t_{i_1} \ldots t_{i_k} \in Perm(f)$ and $\exists h_j \in ((\alpha(P) \cap T_2) \setminus \cup_{q=j}^{k} \{t_{i_q}\})^*$, $j = 1, \ldots, k$ with $M[h_1 t_{i_1} \ldots h_k t_{i_k} >_{GSPN}$ and $M'[h_1 \ldots h_k >_{GSPN}$ (see Fig. 13).

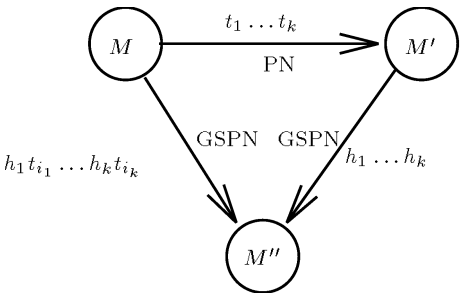

Fig. 13. Illustration of lemma

Proof. The firing sequences $\tilde{f}$ and $h_j, j = 1, \dots, k$ can be constructed as follows:

Set $\tilde{f} := \varepsilon; h_j := \varepsilon, \forall j = 1, \dots, k; \tilde{M} := M$ and $r := 1$.
while $f \neq \varepsilon$ do
  if $\exists t \in f : \tilde{M}[t >_{GSPN}$
  then choose $t \in f$ with $\tilde{M}[t >_{GSPN}$ such that
    (*) $\exists f_1, f_2 \in T^* : f = f_1 t f_2$ and $\forall t' \in f_1 : {}^\bullet t \cap {}^\bullet t' = \emptyset$
      Set $f := f - t; r := r + 1; \tilde{f} := \tilde{f} t; \tilde{M} := Succ(\tilde{M}, t)$
  else choose $t \in \alpha(P)$ with $\tilde{M}[t >_{GSPN}$
    Set $h_r := h_r t; \tilde{M} := Succ(\tilde{M}, t)$

where $Succ(\tilde{M}, t)$ is the marking obtained after firing t at $\tilde{M}$. We will show that the following properties hold after each iteration of the while-loop:

a) $M[f h_1 \dots h_r >_{PN}$
b) $f \neq \varepsilon \Longrightarrow \exists t \in f : \tilde{M}[t >_{PN}$
c) $h_r \in ((\alpha(P) \cap T_2) \setminus \cup_{q=r}^{k} \{t_{i_q}\})^*$

First let us note the following:

1. Since the GSPN is an EC-net satisfying condition EQUAL-Conflict we have $EN_T(\tilde{M}) \cap \alpha(P) \neq \emptyset$.
2. It there is a transition $t \in f$ with $\tilde{M}[t >_{GSPN}$ then all transitions in conflict with t are also GSPN-enabled. Furthermore we have
$$\forall t' \in h_r, t \in \{t_{i_r}, \dots, t_{i_k}\} : {}^\bullet t' \cap {}^\bullet t = \emptyset. \tag{3}$$
3. $h_r \in (\alpha(P) \setminus \cup_{q=r}^{k} \{t_{i_q}\})^*, \forall r = 1, \dots, k$ follows directly from the construction.

ad a) From construction we have $M[t_1 \dots t_k >_{PN}$ and $M[h_1 t_{i_1} \dots h_k t_{i_k} >_{PN}$. We will show $M[t_1 \dots t_k h_1 \dots h_r >_{PN}$ by induction on r.
Base: $r = 1$. $M[t_1 \dots t_k >_{PN}$ and $M[h_1 >_{PN}$ imply $M[t_1 \dots t_k h_1 >_{PN}$ by lemma 16c)[1]
Step: Assuming $M[t_1 \dots t_k h_1 \dots h_r >_{PN}$ we have to show $M[t_1 \dots t_k h_1 \dots h_r h_{r+1} >_{PN}$. Since $h_1 t_{i_1} \dots h_r t_{i_r} \in Perm(t_1 \dots t_k h_1 \dots h_r - t_{i_{r+1}} \dots t_{i_k})$ and $M[h_1 t_{i_1} \dots h_r t_{i_r} h_{r+1} >_{PN}$ property 2 and lemma 16c)[2] yield $M[t_1 \dots t_k h_1 \dots h_{r+1} >_{PN}$.

[1] Choose $f = g = t_1 \dots t_k$ and $q = h_1$.
[2] Choose $f = t_1 \dots t_k h_1 \dots h_r, g = t_{i_{r+1}} \dots t_{i_k}$ and $q = h_{r+1}$.

ad b) Assume $f \neq \varepsilon$. Define

$$s_r := min(\{1, \ldots, k\} \setminus \cup_{p=1}^{r-1}\{i_p\}), r = 1, \ldots, k$$

From this definition we directly get $s_{r+1} \geq s_r$ and

$$\cup_{j=1}^{s_r-1}\{j\} \subseteq \cup_{j=1}^{r-1}\{i_j\}$$

i.e. the set $\{1, \ldots, s_r - 1\}$ is a subset of $\{i_1, \ldots, i_{r-1}\}$.
Thus $t_1 \ldots t_{s_r-1} \in h_1 t_{i_1} \ldots t_{i_{r-1}} h_r$ using the convention $t_k \ldots t_j := \varepsilon$ if $k > j$.
Property 2 gives $\forall t' \in h_1 \ldots h_r : {}^\bullet t' \cap {}^\bullet t_{s_r} = \emptyset$ and (*) additionally implies
$\forall t' \in (t_{i_1} \ldots t_{i_{r-1}} - t_1 \ldots t_{s_r-1}) : {}^\bullet t' \cap {}^\bullet t_{s_r} = \emptyset$.
Since $t_1 \ldots t_{s_r-1} \in h_1 t_{i_1} \ldots t_{i_{r-1}} h_r$ lemma 16c)[3] gives $M[h_1 t_{i_1} \ldots h_r t_{s_r} >_{PN}$ and so $\tilde{M}[t_{s_r} >_{PN}$. Since $t_{s_r} \in f$, we have b).

ad c) It remains to show that $h_r \in T_2^*$. From property b) we get $\tilde{M}[t >_{PN}$ for some $t \in f$. There are two cases to be considered:

c1) $t \in T_2$. Then also $\tilde{M}[t >_{GSPN}$ and $h_r = \varepsilon \in T_2^*$.

c2) $t \in T_1$. Since transitions of T_2 are not in conflict with t (condition EQUAL-Conflict) t remains PN-enabled. If some transition of T_1 gets GSPN-enabled then also t is GSPN-enabled. Thus $h_r \in T_2^*$.

According to lemma 15 the while-loop eventually terminates and since $M[t_1 \ldots t_k h_1 \ldots h_k >_{PN}$ and $h_r \in T_2^*, \forall r \in \{1, \ldots, k\}$ we finally get $M'[h_1 \ldots h_k >_{GSPN}$ which accomplishes the proof. □

Employing lemma 17 the proofs of the following theorems are straight-forward.

Theorem 18. *Let GSPN be an EC-GSPN whose underlying Petri net is live and bounded. Then*
Condition EQUAL-Conflict $\Longleftrightarrow$ *GSPN is live.*

Proof.

"$\Longrightarrow$": Choose an arbitrary transition $t \in T$ and an arbitrary marking $M \in R(GSPN, M_0) \subseteq R(PN, M_0)$. Since PN is live there exists $f \in T^*$ with $M[f >_{PN}$ and $t \in f$. Lemma 17 states the existence of a firing sequence $\sigma \in Perm(fh)$ where $h \in T_2^*$ and $M[\sigma >_{GSPN}$. Since $t \in \sigma$ liveness of the GSPN follows.

"$\Longleftarrow$": Assume that condition EQUAL-Conflict does not hold,
i.e. $\exists t, t' \in T : {}^\bullet t \cap {}^\bullet t' \neq \emptyset$ and $\{t, t'\} \not\subseteq T_1$ and $\{t, t'\} \not\subseteq T_2$.
Since the GSPN is an EC-GSPN, obviously t and t' can not be both live, contradicting the liveness of the GSPN.

□

Theorem 19. *Let GSPN be an EC-GSPN whose underlying Petri net is live and bounded. Then*
Condition EQUAL-Conflict $\Longrightarrow$ *GSPN has home states.*

Proof. (cf. Fig. 12) As underlined before we have to show the directedness property for the GSPN, i.e.

$$R(GSPN, M) \cap R(GSPN, M') \neq \emptyset, \forall M, M' \in R(GSPN, M_0).$$

So choose two arbitrary markings $M, M' \in R(GSPN, M_0)$. Since live and bounded EC-nets have home states, see [18], $\exists M'' \in R(PN, M) \cap R(PN, M')$. Let $f, g \in T^*$ be two

[3] Choose $f = h_1 t_{i_1} \ldots t_{i_{r-1}} h_r, g = h_1 \ldots h_r (t_{i_1} \ldots t_{i_{r-1}} - t_1 \ldots t_{s_r-1})$ and $q = t_{s_r}$.

firing sequences with $M[f >_{PN} M''$ and $M'[g >_{PN} M''$. Lemma 17 implies the existence of $h \in T^*$ and $\tilde{g} \in Perm(gh)$ such that $M''[h >_{GSPN} \tilde{M}'$ and $M'[\tilde{g} >_{GSPN} \tilde{M}'$ for some marking $\tilde{M}' \in R(GSPN, M_0)$.

Since $M[fh >_{PN} \tilde{M}'$ the same argumentation applies to the firing sequence fh. So by lemma 17 there exists $\tilde{h} \in T^*$ and $\tilde{f} \in Perm(fh\tilde{h})$ with $\tilde{M}'[\tilde{h} >_{GSPN} \tilde{M}''$ and $M[\tilde{f} >_{GSPN} \tilde{M}''$ for some marking $\tilde{M}'' \in R(GSPN, M_0)$.

Thus $\tilde{M}'' \in R(GSPN, M) \cap R(GSPN, M')$ which completes the proof. □

7 Conclusions

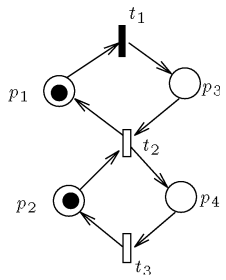

Fig. 14. M_0 is not a home state

We have presented a restriction (condition EQUAL-Conflict) on the timing constraints of an equal conflict GSPN, such that liveness and the existence of home states of the Petri net carry over to the time-augmented net. The restriction can obviously be checked very efficiently and hence it does not affect the overall complexity of qualitative analysis using PN algorithms. Concerning liveness and the existence of home states in combination this restriction is necessary and sufficient thus supporting a combined qualitative and quantitative analysis of GSPNs (cf. Fig. 2). In [4] it is shown that condition EQUAL-Conflict is also important in the context of Queueing Petri nets, an extension of coloured GSPNs.

The results presented in this article are also fundamental for the development of efficient quantitative analysis techniques of GSPNs (cf. [11] for SPNs), since before heading, e.g., for product form representations of the steady state distribution one first has to show its existence.

It is important to note that we have proved that condition EQUAL-Conflict is relevant for EC-nets concerning the properties 'absence of timeless traps', 'liveness' and 'the existence of home states'. It is very difficult to find similar restrictions for general net structures and/or other properties.

E.g., the nets depicted in Fig. 3, 5 and 8 satisfy condition EQUAL-Conflict, but the mentioned qualitative properties of the underlying PN do not hold for the GSPN. Extending condition EQUAL-Conflict for general nets is not trivial, see [3].

So one is at least tempted to assume that condition EQUAL-Conflict is essential for EC-nets concerning also other qualitative properties. But even if the net structure is extremely simple this might not be the case.

E.g., a marking M is a home state of a live and bounded EFC-net iff M marks all traps [6]. This property does not even hold for GSPNs with a marked graph structure satisfying condition EQUAL-Conflict. Figure 14 depicts such a GSPN where M_0 marks all traps, but is not a home state of $R(GSPN, M_0)$. Thus M_0 is not a recurrent state with reference to the Markov chain of the GSPN. We conjecture that this characterisation of home states holds

for tangible markings, i.e. for all markings $M \in R(GSPN, M_0)$ with $EN_T(M) \subseteq T_1$ we have "M is a home state iff M marks all traps".

The last example shows that developing efficiently testable restrictions on the timing constraints is sensitive to the particular property even for EC-nets. In this article we have concentrated on qualitative properties being essential for a quantitative analysis. We think that the analysis procedure proposed in Fig. 2 can be extended to other qualitative properties and/or net classes by taking the PN behaviour partly into account, also giving efficiently testable restrictions on the possible forms of integrating time into nets.

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