On Non-Ergodic Infinite-State Stochastic Petri Nets

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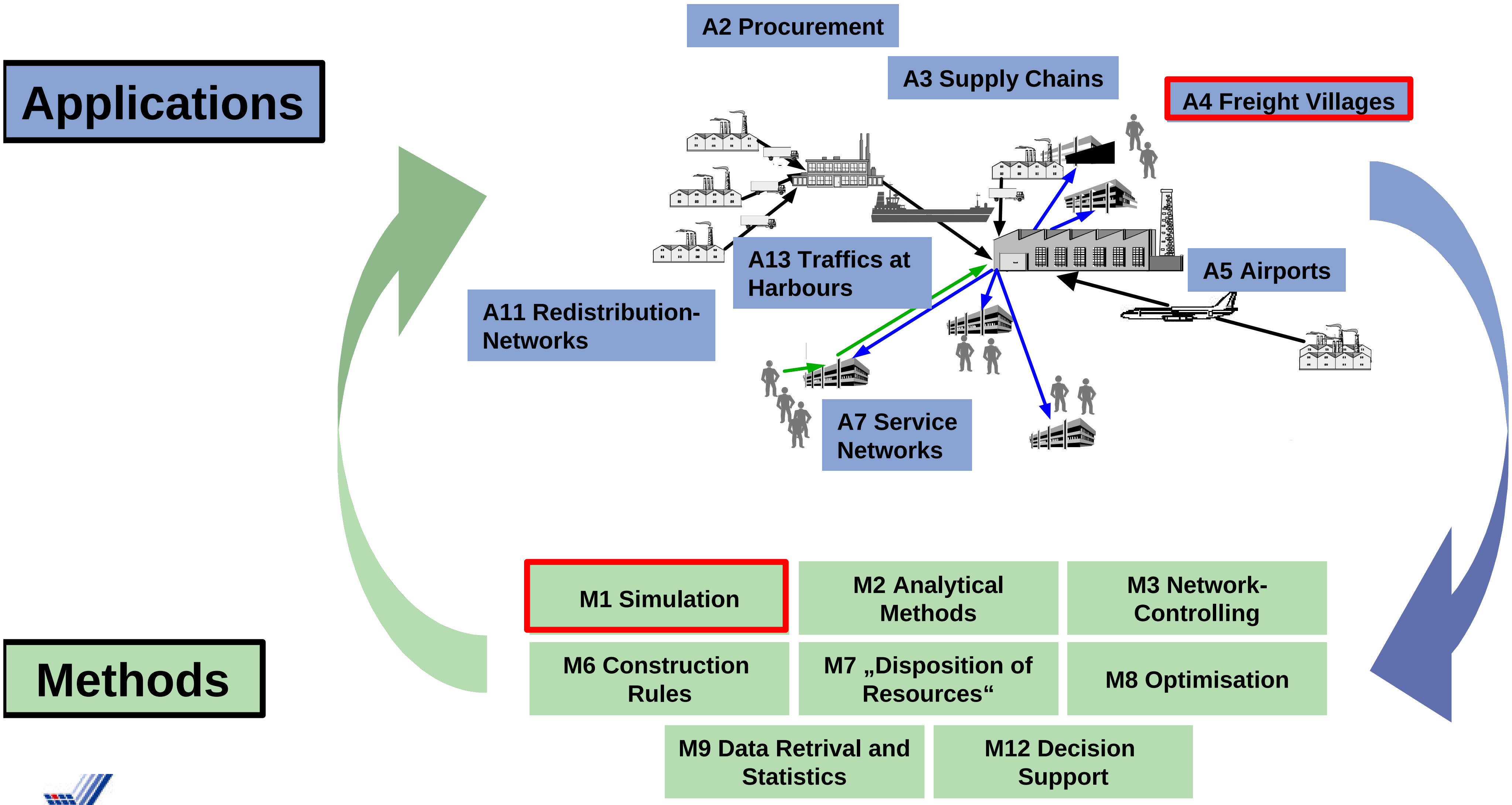
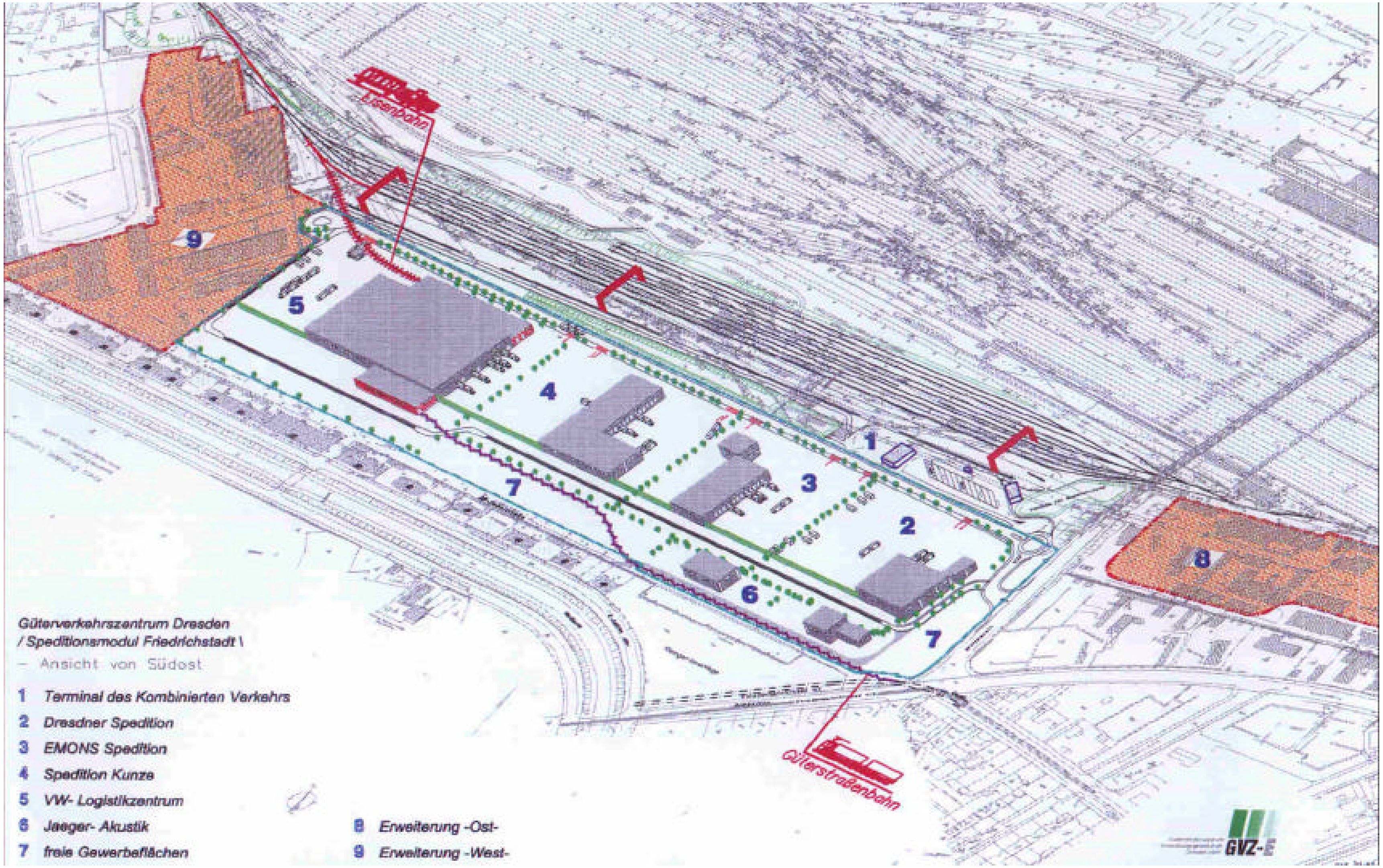
- Motivation
- E-sensitive GSPNs
- Detecting e-sensitive GSPNs
- E-sensitivity and non-ergodicity
- Conclusions

SFB = **S**onder**f**orschungs**b**ereich
= Collaborative Research Center

SFB 559 Modelling of Large Logistics Networks

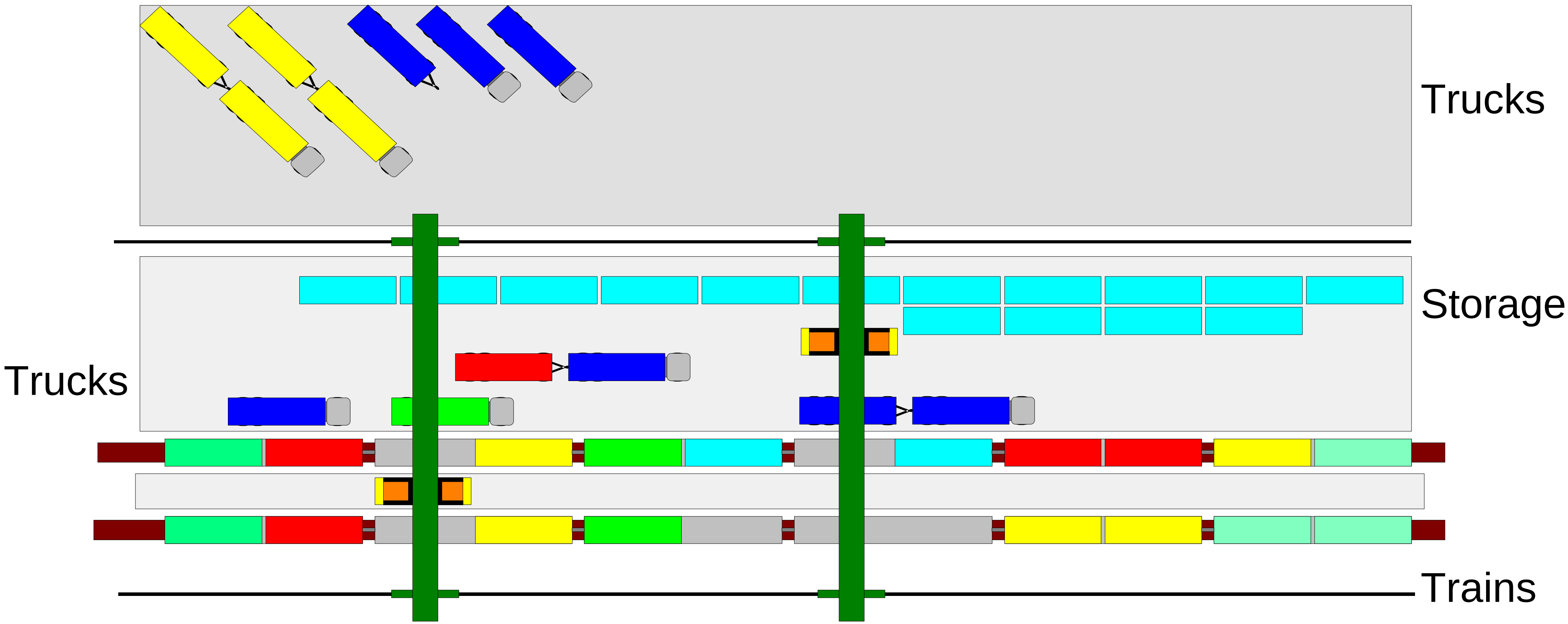


Example: Freight Village



Example: Freight Village

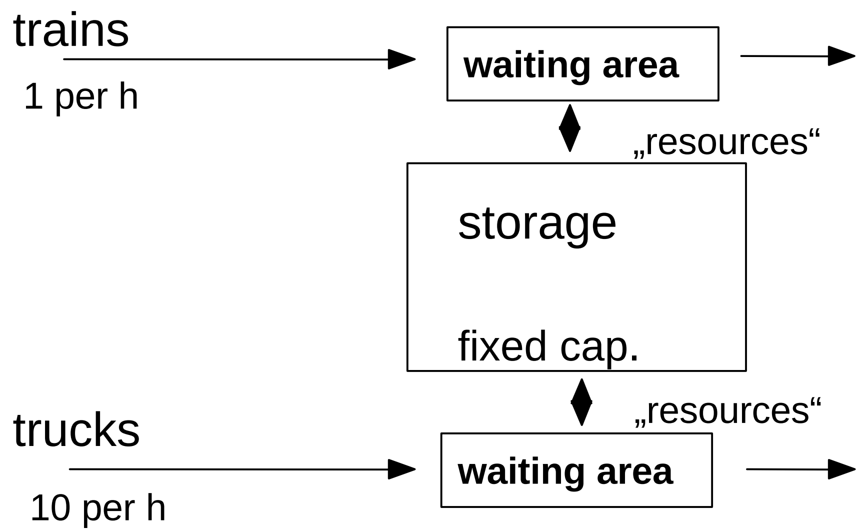
Terminal of a Freight Village



Example: Freight Village

Terminal of a Freight Village

Simplified Model („top-level“)



design phase:

estimation of volume of traffic, e.g. mean number of trains and trucks.

general analysis objective (here):

dimension resources

trains:

determine unload volume UV (uniform [50,150])

determine load volume LV (uniform [50,150])

wait until ((UV + storage) <= storage capacity)

unload

wait until ((storage - LV) >= 0)

load

leave terminal

trucks:

... UV (uniform [5,15])

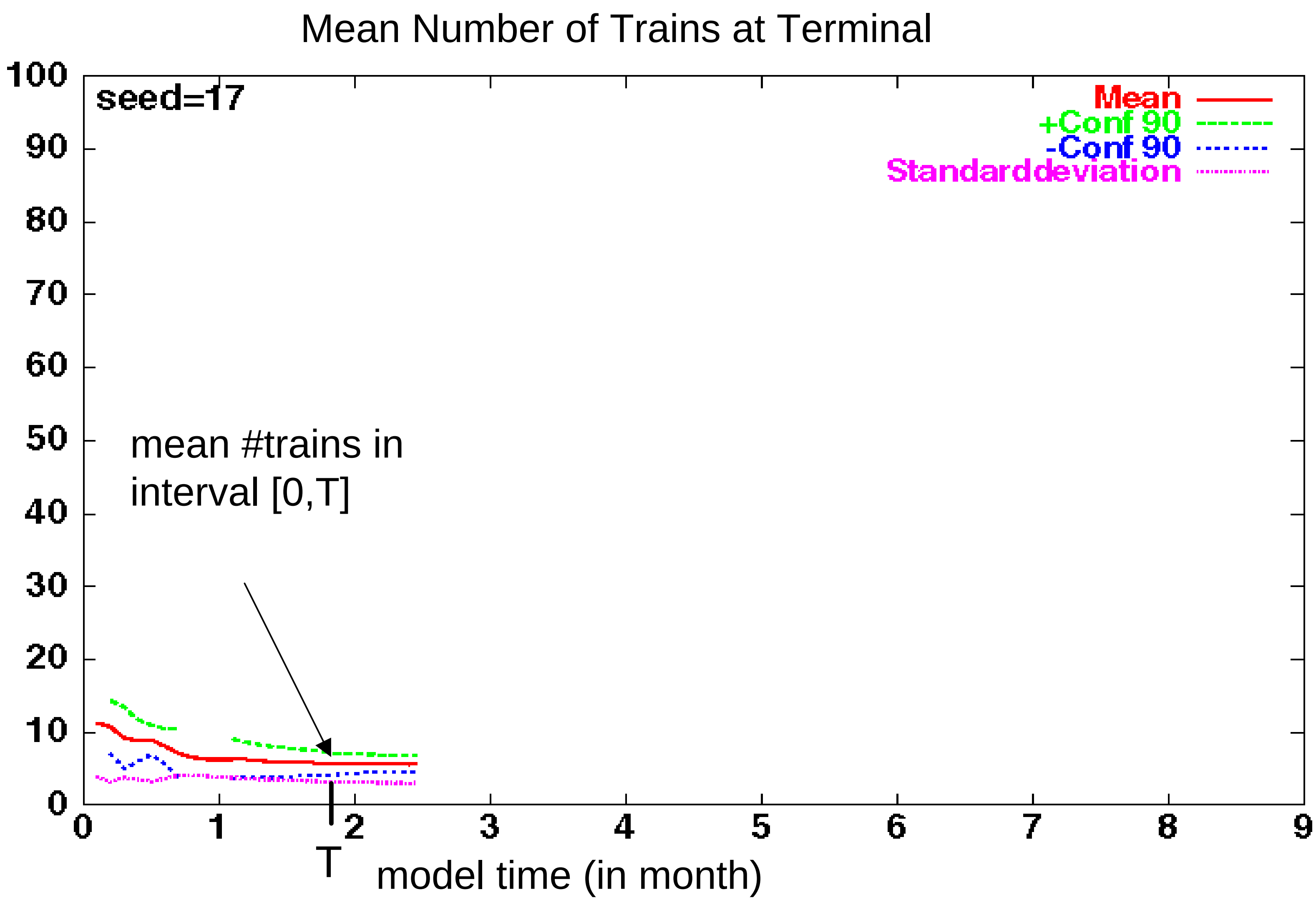
... LV (uniform [5,15])

...

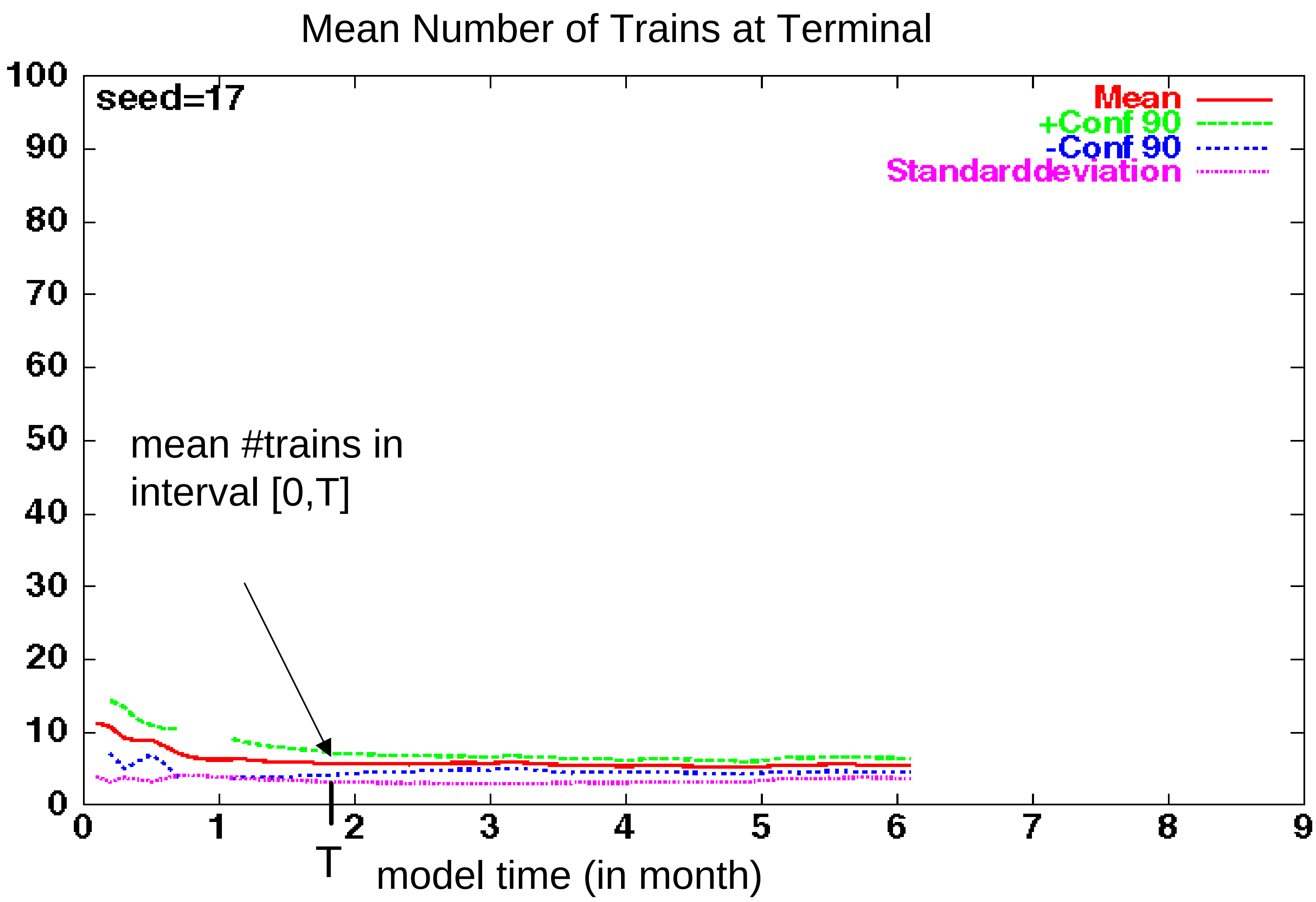
Specific Analysis Objective:

Size of Waiting Areas?

Experiment: Infinite Waiting Areas

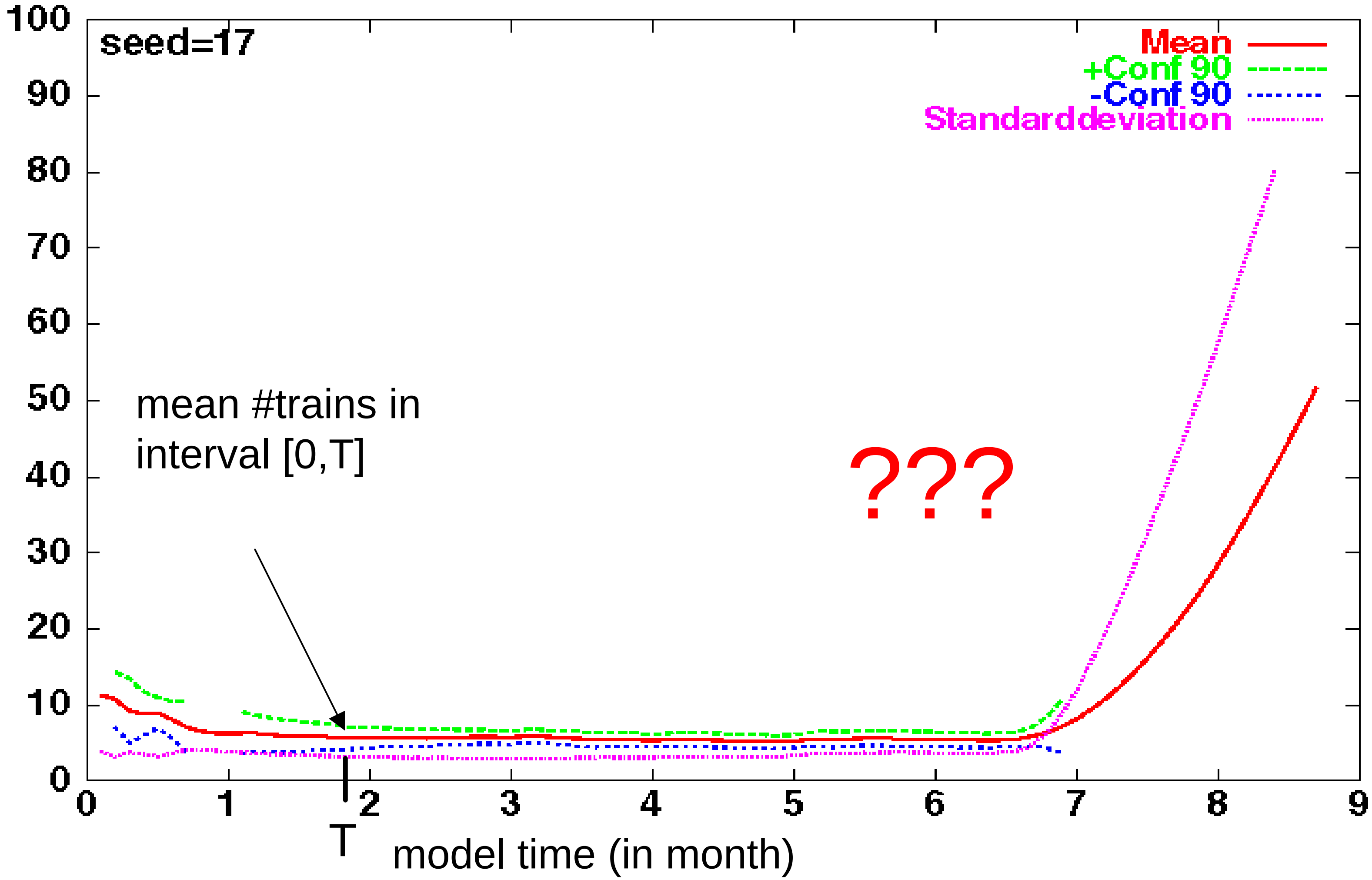


Experiment: Infinite Waiting Areas



Experiment: Infinite Waiting Areas

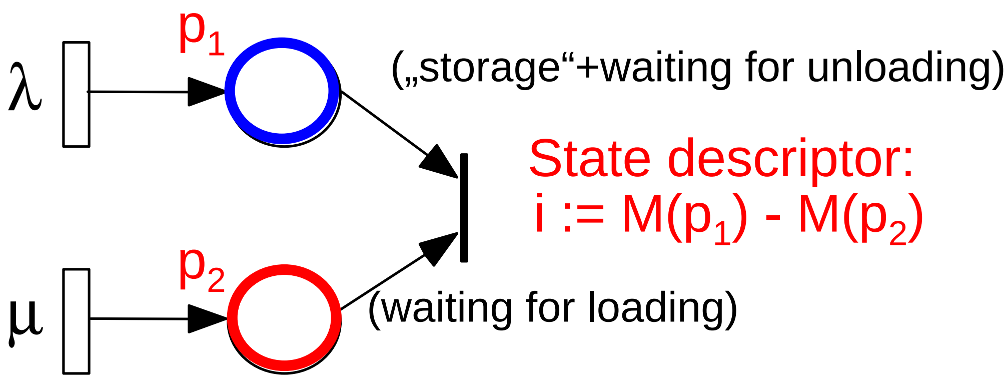
Mean Number of Trains at Terminal



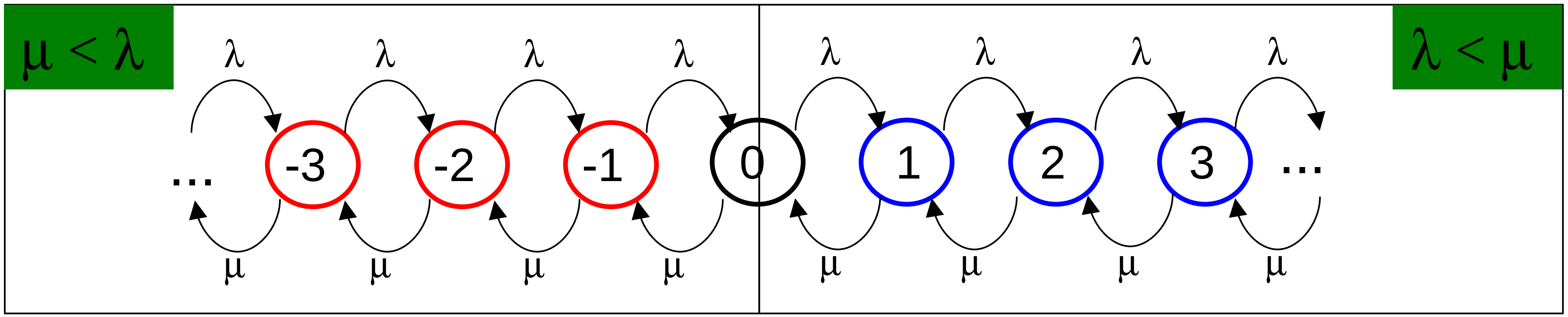
Analytical Investigation:

Simplifications:

- only one type of carrier
- each carrier either unloads or loads just 1 storage unit
- Poisson arrival streams
  with parameters l („unload“) and m („load“)
- all other activities (parking+unloading/loading+leaving)
  are viewed as an elementary, immediately occurring step



Markov model has
NO steady-state distribution
for <u>all</u> λ,μ!



GSPN Definition

A Petri net (PN) is a 4-tuple
$PN = (P, T, C^-, C^+)$ with
$P = \{p_1, \ldots, p_n\}, T = \{t_1, \ldots, t_m\}$,
$P \cap T = \emptyset$,
$C^- = (c^-_{ij}), C^+ = (c^+_{ij}) \in \mathbb{N}_0^{n \times m}$.

C^+: forward incidence matrix
("arcs from t to p")
C^-: backward incidence matrix
("arcs from p to t")
Incidence matrix:
$C := C^+ - C^-$

A GSPN is a tuple
$GSPN = ((PN, M_0), T_1, T_2, \Gamma)$ where

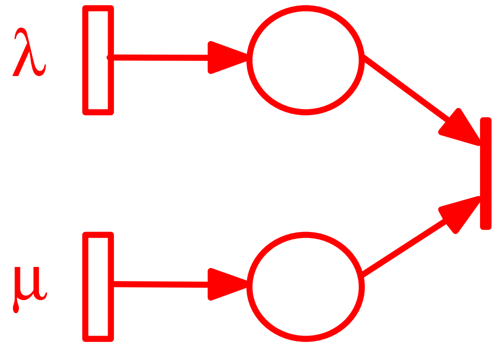
- $PN = (P, T, C^-, C^+)$ is the underlying Petri net,
- M_0 is the initial marking,
- $T_1 \subseteq T$ is the set of timed transitions, $T_1 \neq \emptyset$,
- $T_2 \subset T$ denotes the set of immediate transitions,
  $T_1 \cap T_2 = \emptyset$, $T = T_1 \cup T_2$,
- $\Gamma = (\gamma_1, \ldots, \gamma_{|T|})$ is a vector with components
  $\gamma_i \in \mathbb{R}^+ := \{x | x \in \mathbb{R}, x > 0\}$, where γ_i
  - is the rate of a negative exponential distribution specifying the firing delay, if $t_i \in T_1$, or
  - is a firing weight, if $t_i \in T_2$.

Marking at time z : $M(z)$

Firing vector at time z : $N(z)$

- The <u>marking process</u> is ergodic iff
  $$\exists M^* \in (\mathbb{R}_0^+)^{|P|} : M^* = \lim_{z\to\infty} E[M(z)].$$
  M^* steady-state mean marking.

- The <u>firing process</u> is ergodic iff
  $$\exists N^* \in (\mathbb{R}_0^+)^{|T|} : N^* = \lim_{z\to\infty} \frac{E[N(z)]}{z}.$$
  N^* mean firing flow vector.



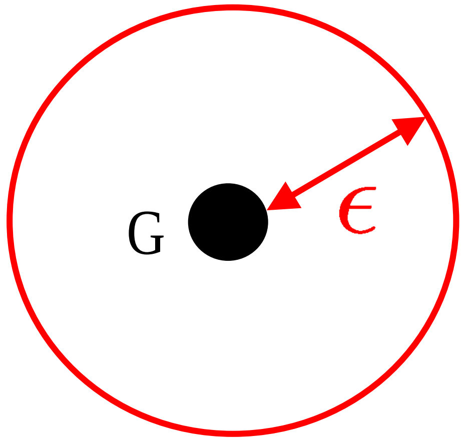
If the marking <u>and</u> the firing processes are ergodic then $\boxed{C \times N^* = 0}$

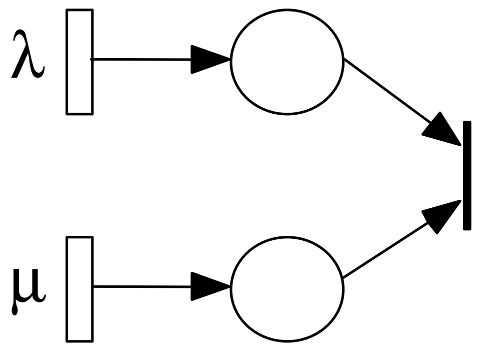
(Florin, Natkin)

Net structure gives:
GSPN has ergodic marking and firing processes
➔ $\lambda = \mu$
➔ Conditon is sensitive towards (even small) changes of parameters

E-sensitive GSPNs

$\Gamma \in \mathbb{R}^m, \epsilon \in \mathbb{R}^+$. $(|T| = m)$
$\Gamma + \bar{\epsilon}_m$ is the set of vectors, which are in an ϵ-area around the point $\Gamma \in \mathbb{R}^m$.





$\Gamma = (\lambda, \mu, 1)$

$\Gamma' = (\lambda, \mu + \frac{\epsilon}{2}, 1)$

A $GSPN = ((PN, M_0), T_1, T_2, \Gamma)$ is called e-insensitive iff there exists $\epsilon \in \mathbb{R}^+$ such that $GSPN(\Gamma') := ((PN, M_0), T_1, T_2, \Gamma')$ has ergodic marking and firing processes for all $\Gamma' \in \Gamma + \bar{\epsilon}_m$, the GSPN is called **e-sensitive** otherwise.

E-sensitive GSPNs

$$C \times N^* = 0$$

- Let GSPN be e-insensitive. Then

$$\exists \epsilon \in \mathbb{R}^+ : \forall \Gamma' \in \Gamma + \bar{\epsilon}_m : \qquad C \times N^*(\Gamma') = 0$$

  where $N^*(\Gamma')$ denotes the mean firing flow vector of GSPN(Γ').

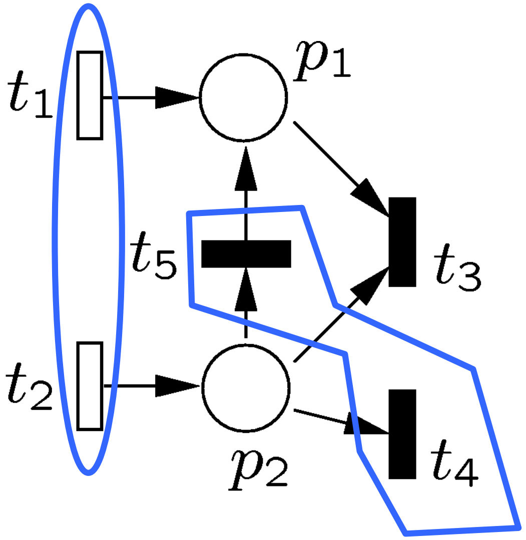
- If

$$\forall \epsilon \in \mathbb{R}^+ : \exists \Gamma' \in \Gamma + \bar{\epsilon}_m : \qquad C \times N^*(\Gamma') \neq 0$$

  then the GSPN is e-sensitive.

Problem: Determine $N^*(\Gamma')$.

Partial Equal Conflict (PEC)



$$N_1^* = \gamma_1$$
$$N_2^* = \gamma_2$$
$$\frac{N_4^*}{N_5^*} = \frac{\gamma_4}{\gamma_5}$$

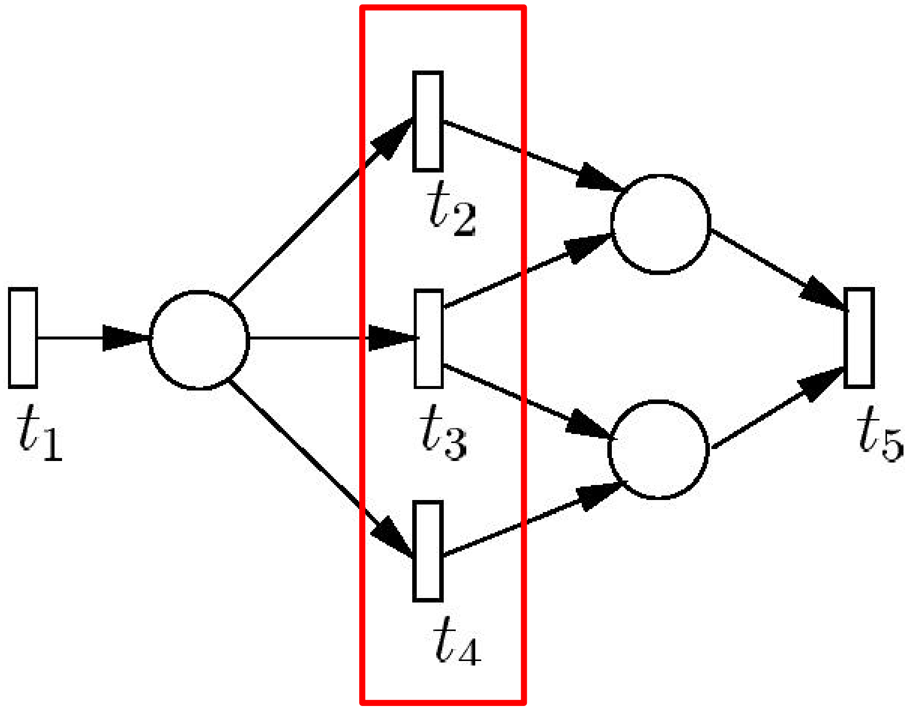
$$N_4^* \gamma_5 = N_5^* \gamma_4$$
$$N_1^* \gamma_2 = N_2^* \gamma_1$$

All transitions in $\tilde{T}$ are either timed or immediate

A set of transitions $\tilde{T} \subseteq T, \tilde{T} \neq \emptyset, \tilde{T} \subseteq T_1$ or $\tilde{T} \subseteq T_2$, is in Partial Equal Conflict (PEC) iff

$$\forall t_i, t_j \in \tilde{T} : C^-(\bullet, i) = C^-(\bullet, j)$$

C^- backward incidence matrix (”arcs from p to t”)



$\{t_2, t_3, t_4\}$ in PEC

$$N_2^* \gamma_3 = N_3^* \gamma_2$$
$$N_3^* \gamma_4 = N_4^* \gamma_3$$
$$N_2^* \gamma_4 = N_4^* \gamma_2$$

N_2^*, N_3^*, N_4^* are dependent!

$C \times N^* = 0$ N^* is in kernel(C)

Basis for kernel C:

[1,0,1,0,1],
[2,1,0,1,1]

Rank of submatrix
(= columns of kernel C for t_2, t_3, t_4)
= 2
< number of elements in set $\{t_2, t_3, t_4\}$

Rank of submatrix < |PEC set|
gives (here):
N_2^*, N_3^*, N_4^* are dependent!

Detecting e-sensitive GSPNs

Projection of v on $\tilde{T}$ is denoted by $Proj(v, \tilde{T}) \in \mathbb{R}^m$ and defined by

$$(Proj(v, \tilde{T}))_i := \begin{cases} v_i & \text{if } t_i \in \tilde{T} \\ 0 & \text{otherwise} \end{cases}$$

"projection gives submatrix"

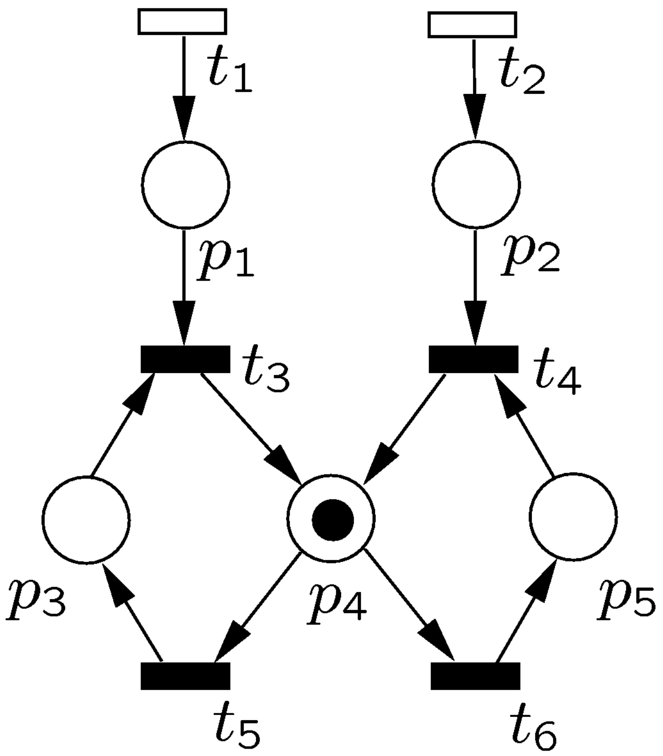
Given a GSPN with ergodic firing process and $N^* = \lim_{z \to \infty} \frac{E[N(z)]}{z}$.
Let $k_1, \ldots, k_r, k_i \in \mathbb{R}^m, k_i \neq 0$, be a basis of kernel($C$).
If there exists $\tilde{T} \subseteq T$ with $\tilde{T}$ in PEC and

$$\text{rank}\left(\left(Proj(k_1, \tilde{T}) \ldots Proj(k_r, \tilde{T})\right)\right) < |\tilde{T}|$$

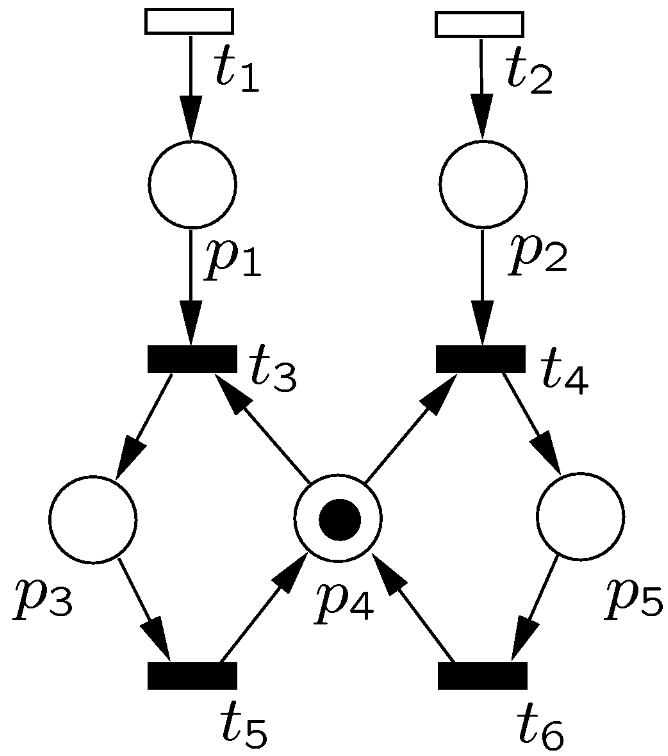
then the GSPN is e-sensitive or $Proj(N^*, \tilde{T}) = 0$.

polynomial complexity, only maximal PEC sets need to be tested!
(T' in PEC, $\tilde{T} \subseteq T' \Rightarrow \tilde{T}$ in PEC.)

Limits



E-sensitive (non-ergodic)



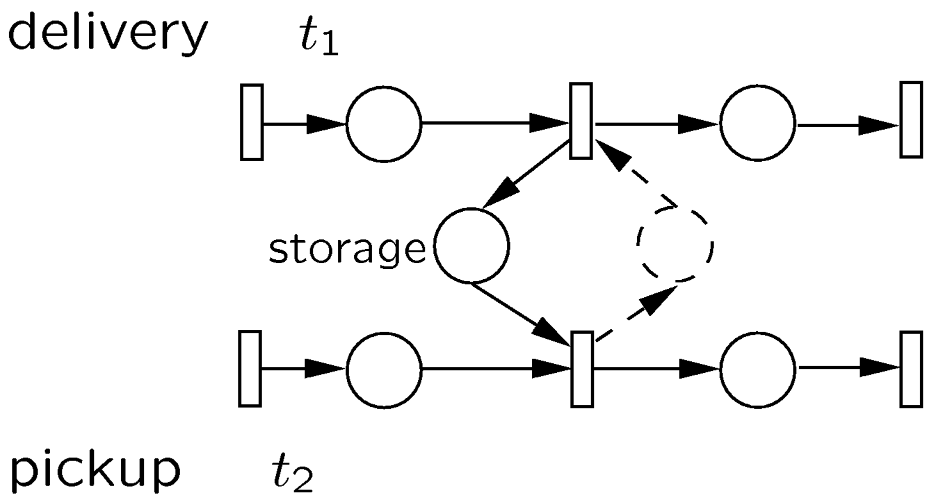
E-insensitive (ergodic)

(„M/M/1 two classes“)

Same kernel(C)!

Applications

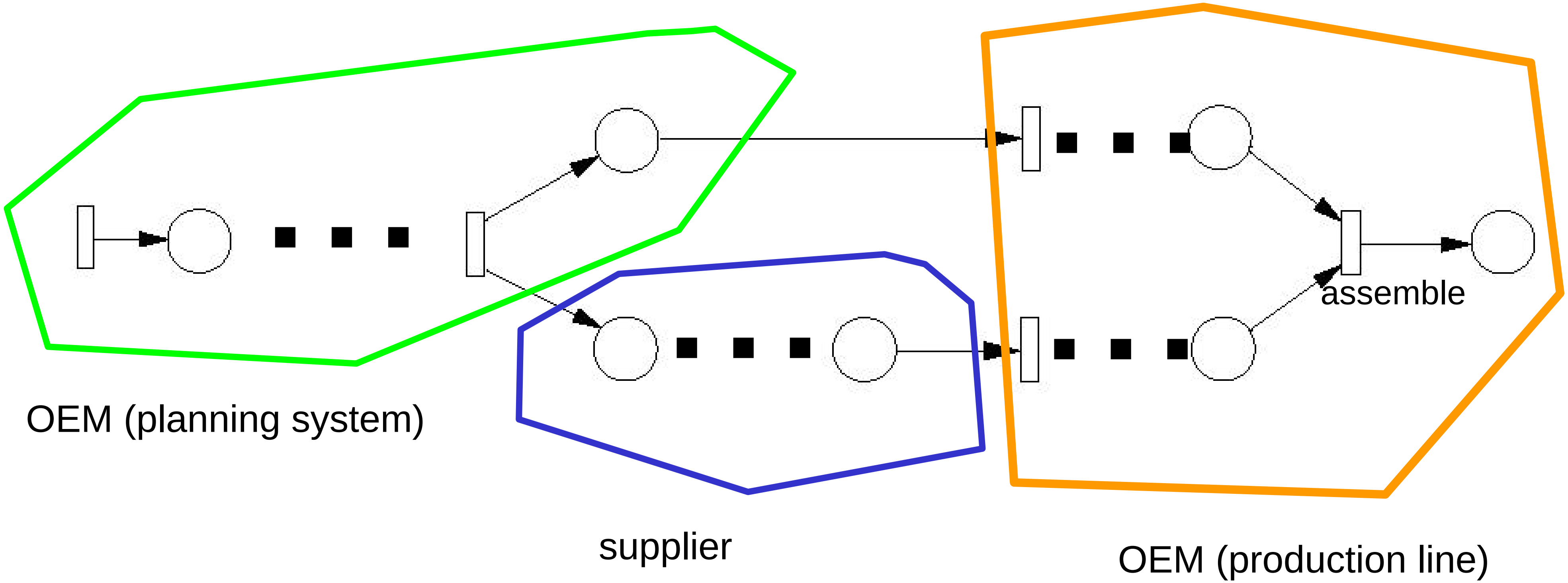
- stockkeeping scenario:



Basis for kernel(C) $= \{(1,1,1,1,1,1)\}$
$\{t_1, t_2\}$ in PEC.
$\Rightarrow$ GSPN is e-sensitive.

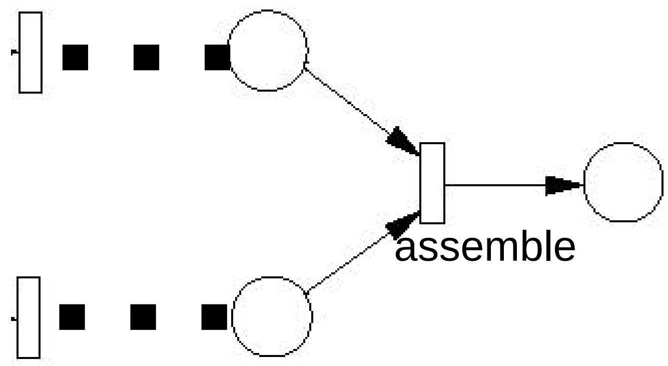
Applications (cont‘d)

- procurement channel:



Applications (cont'd)

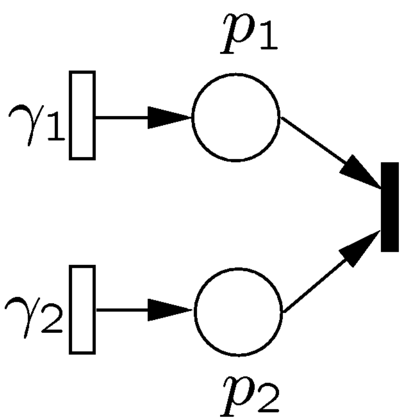
- procurement channel:



$\Rightarrow$ GSPN is e-sensitive.

Does e-sensitivity imply non-ergodicity?

Consider GSPN with marking-dependent (+ parameter x) firing rates:



$i := M(p_1) - M(p_2)$ state descriptor

$$\gamma_1(x, i) = e^{\frac{1}{2}(V(x,i)-V(x,i+1))}$$

$$\gamma_2(x, i+1) = e^{\frac{1}{2}(V(x,i+1)-V(x,i))}$$

$$V(x, i) := |i| - x^2 i^2$$

$$\pi(x, i) = \frac{e^{-V(x,i)}}{\sum_{j=-\infty}^{\infty} e^{-V(x,j)} < \infty \text{ iff } x = 0}$$

$x = 0$ gives

$M^* = \frac{e}{e^2-1}(1, 1)$

$N^* = 2\frac{\sqrt{e}}{e+1}(1, 1, 1)$

$\exists$ similar examples for GSPNs with "fixed" firing parameters **?**

Conclusions

- E-sensitivity
- Some e-sensitive GSPNs can be efficiently detected by inspection of the net structure

$$\text{rank}\left(\left(Proj(k_1,\tilde{T})\ldots Proj(k_r,\tilde{T})\right)\right) < |\tilde{T}| \quad \tilde{T} \text{ in PEC}$$

- Open question:
  Does e-sensitivity imply non-ergodicity for GSPNs with „fixed“ firing parameters?

- E-sensitive GSPNs are especially problematic when simulating the net („finite number representation“, pertubed chain)