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Performance Analysis of the General Packet Radio Service

Christoph Lindemann and Axel Thümmler
University of Dortmund
Department of Computer Science
August-Schmidt-Str. 12
44227 Dortmund, Germany
http://www4.cs.uni-dortmund.de/~Lindemann/

Abstract

This paper presents an efficient and accurate analytical model for the radio interface of the General Packet Radio Service (GPRS) in a GSM network. The model is utilized for investigating how many packet data channels should be allocated for GPRS under a given amount of traffic in order to guarantee appropriate quality of service. The presented model constitutes a continuous-time Markov chain. The Markov model represents the sharing of radio channels by circuit switched GSM connections and packet switched GPRS sessions under a dynamic channel allocation scheme. In contrast to previous work, the Markov model explicitly represents the mobility of users by taking into account arrivals of new GSM and GPRS users as well as handovers from neighboring cells. To validate the simplifications necessary for making the Markov model amenable to numerical solution, we provide a comparison of the results of the Markov model with a detailed simulator on the IP level.

1. Introduction

The *General Packet Radio Service (GPRS)* is a standard from the *European Telecommunications Standards Institute (ETSI)* on packet data in GSM systems [3], [5]. By adding GPRS functionality to the existing GSM network, operators can give their subscribers resource-efficient wireless access to external Internet protocol-based networks, such as the Internet and corporate intranets. The basic idea of GPRS is to provide a packet-switched bearer service in a GSM network. As impressively demonstrated by the Internet, packet-switched networks make more efficient use of the resources for bursty data applications and provide more flexibility in general.

To evaluate the performance of GPRS, several detailed simulation studies considering performance of TCP over GPRS [7], [9], dimensioning of the GPRS core network [8] and correlation of GSM and GPRS users for different channel allocation techniques [11] were conducted.

In previous work, several analytical models based on continuous-time Markov chains have been introduced for studying performance issues in GSM networks. Ajmone Marsan, Marano, Mastroianni, and Meo evaluated the impact of reserving channels for data and multimedia services on the performance in a circuit switched GSM network [1]. Boucherie and Litjens developed a Markov model for analyzing the performance of GPRS under a given GSM call characteristic [2]. Recently, Ermel, Begain, Müller, Schüler, and Schweigel developed a Markov model for deriving blocking probabilities and average data rates for GPRS in GSM networks [4]. In none of these previous work, the question how many packet data channels should be allocated for GPRS for a given amount of traffic in order to guarantee appropriate quality of service has been investigated.

This paper presents an efficient and accurate analytical model for the radio interface of the General Packet Radio Service (GPRS) in a GSM network. The presented model constitutes a continuous-time Markov chain. The Markov model introduced in this paper represents the sharing of radio channels by circuit switched GSM connections and packet switched GPRS sessions under a dynamic channel allocation scheme. We assume a fixed number of physical channels permanently reserved for GPRS sessions and the remaining channels to be shared by GSM and GPRS connections. The model is utilized for investigating how many packet data channels should be allocated for GPRS for a given amount of traffic in order to guarantee appropriate quality of service. We present performance curves for average carried data traffic, packet loss probability, and throughput per user, for different network configurations.

In contrast to previous work, the Markov model explicitly represents the mobility of users by taking into account arrivals of new GSM and GPRS users as well as handovers from neighboring cells. Furthermore we employ a Markov modulated Poisson process for modeling arrivals of data packets in order to take into account the burstiness of IP traffic generated from GPRS sessions. We consider a cluster comprising of seven hexadiagonal cells in an integrated GSM/GPRS network, serving circuit-switched voice and packet-switched data sessions. To allow the effective employment of numerical solution methods, the Markov model represents just one cell (i.e., the mid cell) and employs the procedure for balancing incoming and outgoing handover rates introduced in [1]. To validate this simplification, we provide a comparison of the results of the Markov model with a detailed simulator implemented using the simulation library CSIM [10]. This validation shows that almost all performance curves derived from the Markov model lie in the confidence intervals of the corresponding curve of the simulator.

Because of the employment of a numerical method for steady-state analysis, we can efficiently and accurately compute sensitive performance measures such as loss probabilities. In fact, using the presented Markov model sensitive performance measures can be computed on a modern PC within some minutes of CPU solution time and with numerical accuracy of 10^{-15}.

The remainder of the paper is organized as follows. Section 2 describes the basic GPRS network architecture and the radio interface which provide the technical background of the Markov model. In Section 3, we derive the Markov model and introduce its parameters. Comprehensive performance studies for GPRS are presented in Section 4. A detailed comparison of the performance between different network configurations and percentages of GPRS users is provided.

2. General Packet Radio Service

The introduction of GPRS as an additional service to GSM networks requires several modifications to the network architecture. Some of the nodes already implemented in current GSM systems can be shared between GPRS and GSM. Two new node types, *Serving GPRS Support Node (SGSN)* and *Gateway GPRS Support Node (GGSN)*, have to be introduced. Figure 1 depicts the network architecture of GPRS in GSM networks. The GGSN is the gateway node between an external packet-switched data network (e.g. IP, X.25) and the GPRS core network. Its task is to assign the correct SGSN for a *Mobile Stations (MS)* depending on the location of the MS. The SGSN connects the GPRS backbone and the radio access network, and switches the packets to the correct *Base Station Controller (BSC)* via the Gb interface. The *Base Transceiver Station (BTS)* is only a relay station without protocol functions. It performs the modulation of the carrier frequencies and demodulation of the signals.

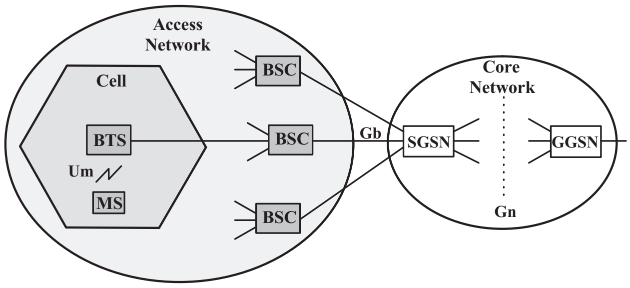

Figure 1. Basic GPRS Network Architecture

On the physical layer, GSM uses a combination of *Frequency Division Multiple Access (FDMA)* and *Time Division Multiple Access (TDMA)* for multiple access. Two frequency bands are reserved for GSM operation, one for transmission from the mobile station to the BTS (uplink) and one for transmission form the BTS to the mobile station (downlink). Each of these bands is divided into 124 single carrier channels of 200 kHz width. A certain number of these frequency channels is allocated to a BTS, i.e., to a cell. Each of the 200 kHz frequency channels is divided into eight time slots that form a TDMA frame. A time slot lasts for a duration of 0.577 ms and carries 114 bits of information. The recurrence of one particular time slot defines a physical channel. Physical channels allocated for GPRS are called *Packet Data Channels (PDCH)*.

The channel allocation in GPRS is different from the original allocation scheme of GSM. GPRS allows a single mobile station to transmit on multiple time slots of the same TDMA frame. This results in a very flexible channel allocation: one to eight time slots per TDMA frame can be allocated to one mobile station. On the other hand a time slot can be assigned temporarily to a mobile station, so that one to eight mobile stations can use one time slot. Moreover, uplink and downlink channels are allocated separately, which efficiently supports asymmetric data traffic.

In conventional GSM, a channel is permanently allocated for a particular user during the entire call period (whether data is transmitted or not). In contrast to this, in GPRS the channels are only allocated when data packets are sent or received, and they are released after the transmission. For bursty traffic this results in a much more efficient usage of the scarce radio resource. With this principle, multiple users can share one physical channel. GPRS includes the functionality to increase or decrease the amount of radio resources allocated to GPRS on a dynamic basis (“capacity on demand”). The PDCHs are taken from the common pool of all channels available in the cell. Physical channels not currently in use by conventional GSM can be allocated as PDCHs to increase the quality of service for GPRS. When there is a resource demand for services with higher priority, e.g. GSM voice calls, PDCHs can be de-allocated.

3. The Markov Model

3.1. Model Description and Parameters

We consider a single cell in an integrated GSM/GPRS network, serving circuit-switched voice and packet-switched data calls. We assume that GSM calls and GPRS calls arrive according to two mutually independent Poisson processes, with arrival rates λ_{GSM} and λ_{GPRS}, respectively. GSM calls are handled circuit-switched, so that one physical channel is exclusively dedicated to the corresponding mobile station. After the arrival of a GPRS call, a *GPRS session* begins. During this time, the *Base Station Controller (BSC)* schedules the radio interface (i.e., the physical channels) among different GPRS users. GPRS user receive packets according to a specified traffic model. The amount of time that a mobile station with an ongoing call remains within the cell is called *dwell time*. If the call is still active after the dwell time, a handover toward an adjacent cell takes place. The *call duration* is defined as the amount of time that the call will be active, assuming it completes without being forced to terminate due to handover failure. We assume the dwell time to be an exponentially distributed random variable with mean $1/\mu_{h,GSM}$ for GSM calls and $1/\mu_{h,GPRS}$ for GPRS sessions. The call durations are also exponentially distributed with mean values $1/\mu_{GSM}$ and $1/\mu_{GPRS}$ for GSM and GPRS sessions, respectively.

To exactly model the user behavior in the cell, we have to consider the handover flow of active GSM calls and GPRS sessions from adjacent cells. It is impossible to specify in advance the intensity of the incoming handover flow. This is due to the fact that the handover rate out of the cell depends on the number of active customers within the cell. On the other hand, the handover rate into the cell depends on the number of customers in the neighboring cells. Thus, the iterative procedure introduced in [1] is employed for balancing the incoming and outgoing handover rates.

Since in the end-to-end data path, the wireless link is typically the performance bottleneck, the Markov model represents the radio interface of an integrated GSM/GPRS network especially the correlation of circuit switched GSM and packet switched GPRS connections. The functionality of the GPRS core network is not included. Because of the anticipated traffic asymmetry, the model focuses on resource contention in the downlink (i.e., the path BSC$\rightarrow$BTS$\rightarrow$MS) of the radio interface. The amount of uplink traffic, e.g. induced by acknowledgments, is assumed to be negligible. The arrival stream of data packets is modeled at the IP layer, assuming a IP packet size of 1 Kbyte. IP packets arriving at the BSC were stored in a FIFO buffer with limited size of K IP packets until they were transmitted on a free physical channel. Let N be the overall number of physical channels available in the cell. We assume that N_{GPRS} channels are permanently reserved as PDCHs for GPRS and the remaining $N_{GSM} = N - N_{GPRS}$ channels can be used either as GSM traffic channels or as "on-demand" PDCHs. Among the on-demand channels, GSM calls have priority. That is on-demand channels allocated as PDCH were immediately released if requested by a GSM call. Throughout this paper, we fix the modulation and coding scheme to CS-2 as explained in [7]. It allows a data transfer rate of 13,4 kbit/sec on one PDCH.

The employed traffic model constitutes a *Markov-modulated Poisson Process (MMPP)* [6]. The MMPP generates IP traffic for each individual GPRS user in the cell. The MMPP has been extensively employed for modeling traffic processes with time-varying arrival rate. Opposed to an ordinary Poisson process, the MMPP can capture some of the important correlations between the interarrival times. In the performance study presented in Section 4, we consider an MMPP controlled by a two-state continuous-time Markov chain. Thus, the MMPP is specified by the infinitesimal generator matrix $\mathbf{Q}$ and rate matrix $\mathbf{\Lambda}$:

$$\mathbf{Q} = \begin{pmatrix} -\alpha & \alpha \\ \beta & -\beta \end{pmatrix}, \qquad \mathbf{\Lambda} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

The two states represent bursty mode and non-bursty mode of the arrival process for IP packets. The arrival process resides in bursty mode for a mean time of $1/\alpha$ and produces arrivals with rate λ_1 and in non-bursty mode for a mean time of $1/\beta$ with arrival rate λ_2 respectively. We characterize such an MMPP by the *average arrival rate* of packets, λ_{avg} and the *degree of burstiness,* B. The former is given by:

$$\lambda_{avg} = \frac{\beta \cdot \lambda_1 + \alpha \cdot \lambda_2}{\alpha + \beta}$$

The degree of burstiness is computed by the ratio between the bursty arrival rate and the average arrival rate, i.e., $B = \lambda_1/\lambda_{avg}$.

3.2. State Definition and Driving Processes

The state of the model representing the considered cell is determined by the number of GSM connections currently active, denoted by n ($0 \le n \le N_{GSM}$), the number of active GPRS sessions, denoted by m ($0 \le m \le M$), the number of packets in the BSC buffer denoted by k ($0 \le k \le K$), and the states r_i of the two-state MMPPs for active GPRS sessions with $0 \le i \le m$. As a consequence, the state of the model can be specified as the vector $s = (n, k, m, r_1, ..., r_M)$ with $r_i = 1$ or $r_i = 2$ for $1 \le i \le m$ and $r_i = 0$ for $m+1 \le i \le M$. This leads to $\left(2^{M+1} - 1\right)\left(N_{GSM} + 1\right)\left(K + 1\right)$ feasible states. Due to its large state space, such a Markov model can be analyzed by discrete-event simulation only. Making the common assumption that all GPRS user behave statistically identical, allows us to derive an aggregated Markov model whose state space is tractable for numerical solution. The rationale behind the aggregation lies in the fact that m identical two-state MMPPs corresponding to m active GPRS sessions can be represented by one MMPP with m+1 states [6]. Employing this aggregation, the state of the Markov model for the cell can be expressed by a vector s = (n, k, m, r). In this tuple, r represents the state of the MMPP for m concurrently active GPRS sessions. The state r of the aggregated MMPP models that r MMPPs of individual GPRS sessions are in non-bursty mode and the remaining m-r MMPPs are in bursty mode. This reduces the state space significantly to an overall number of $\frac{1}{2}(M+1)(M+2)(N_{GSM}+1)(K+1)$ states.

The behavior of GSM users in the considered cell can be represented by an M/M/c/c queue with c = N_{GSM} servers. This is because GSM users are not effected by data traffic of GPRS sessions due to their higher priority. The arrival process of GSM voice calls is the superposition of two Poisson processes corresponding to newly arriving voice calls and incoming handover requests. Therefore, the arrival rate of the M/M/c/c queue is given by $\lambda_{GSM}+\lambda_{h,GSM}$. In the same way, the service rate of the M/M/c/c queue is derived as $\mu_{GSM}+\mu_{h,GSM}$. Moreover, the behavior of GPRS users can be represented in the same way by an M/M/c/c queue with c = M servers and arrival and service rates $\lambda_{GPRS}+\lambda_{h,GPRS}$ and $\mu_{GPRS}+\mu_{h,GPRS}$, respectively. The Markov model constitutes a compound queueing system whose arrival process is governed by the number of active GPRS users (i.e., the customers of the latter M/M/c/c queue) and whose service process is governed by the number of active GSM connections (i.e., the customers of the former M/M/c/c queue).

Let S be the state space of the Markov model we just described. For notational convenience, we enumerate its states from 0 to S_{max}. The model dynamics are determined by the underlying continuous-time Markov chain which cause state transitions at random instants. State transitions correspond to different kinds of events that must be processed in the cell. The following kinds of events may occur:

(i) incoming GSM calls and handovers,

(ii) incoming GPRS sessions and handovers,

(iii) leaving GSM calls due to completion or handover,

(iv) leaving GPRS sessions due to completion or handover,

(v) arrivals of IP packets,

(vi) service of IP packets,

(vii) state changes of the MMPP to a more bursty or less bursty arrival of data packets.

Table 1. Transitions from a state (k, n, m, r)

Event Type	Condition	Successor State	Rate
GSM call arrival	$n < N_{GSM}$	(k, n + 1, m, r)	$\lambda_{GSM} + \lambda_{h,GSM}$
GPRS session arrival	$m < M$	(k, n, m + 1, r)	$\frac{\beta}{\alpha+\beta} \cdot (\lambda_{GPRS} + \lambda_{h,GPRS})$
		(k, n, m + 1, r + 1)	$\frac{\alpha}{\alpha+\beta} \cdot (\lambda_{GPRS} + \lambda_{h,GPRS})$
GSM call leaving cell	$n > 0$	(k, n – 1, m, r)	$n \cdot (\mu_{GSM} + \mu_{h,GSM})$
GPRS session leaving cell	$(m > 0) \wedge (r = 0)$	(k, n, m – 1, r)	$m \cdot (\mu_{GPRS} + \mu_{h,GPRS})$
	$(m > 0) \wedge (r = m)$	(k, n, m – 1, r – 1)	$m \cdot (\mu_{GPRS} + \mu_{h,GPRS})$
	$(m > 0) \wedge (0 < r < m)$	(k, n, m – 1, r – 1)	$\frac{r}{m} \cdot m \cdot (\mu_{GPRS} + \mu_{h,GPRS})$
		(k, n, m – 1, r)	$\frac{m-r}{m} \cdot m \cdot (\mu_{GPRS} + \mu_{h,GPRS})$
Packet arrival	$(k < K) \wedge (m > 0)$	(k + 1, n, m, r)	$(m - r) \cdot \lambda_1 + r \cdot \lambda_2$
Packet service	$\min(N - n, 8k) > 0$	(k – 1, n, m, r)	$\min(N - n, 8k) \cdot \mu_{service}$
Less bursty	$r < m$	(k, n, m, r + 1)	$(m - r) \cdot \alpha$
More bursty	$r > 0$	(k, n, m, r – 1)	$r \cdot \beta$

One can easily show that the continuous-time Markov chain underlying the Markov model has finite state space and is homogeneous and irreducible. Thus, the steady state distribution $\boldsymbol{\pi} = (\pi_0, \pi_1, ..., \pi_{Smax})$ can be computed through the matrix equation $\boldsymbol{\pi} \cdot \mathbf{Q} = 0$ together with the normalization condition $\sum_{i=0}^{S_{max}} \pi_i = 1$. Here, **Q** denotes the infinitesimal generator matrix. The transition rates, i.e. the entries of matrix **Q**, are obtained from the analysis of the system events. For each event, it is possible to determine what state transitions can happen, i.e. what are the possible successor states of a generic state s = (k, n, m, r). This is what we discuss next, referring to Table 1 which shows the conditions on the model state for a transition to be possible, the rate associated with the transition, and the successor state, for each type of events.

Incoming GSM calls and handovers are accepted in the cell if the number of free channels, excluding those reserved as PDCHs, is such that the call can be accommodated. Incoming GPRS sessions and handovers are accepted in the cell if the maximal number of GPRS users M is not reached. A new GPRS session in the cell starts sending packets according to an MMPP described above. We assume the MMPP to start in steady state, that is in bursty-mode with probability $\frac{\beta}{\alpha+\beta}$ and in non-bursty mode with probability $\frac{\alpha}{\alpha+\beta}$. This assumption guaranties that the MMPP is still in steady state when the GPRS session is terminated. Both the completion of calls and the outgoing handover requests have the effect of freeing a channel in the cell. Thus with n active GSM calls the rate of freeing a channel is $n \cdot (\mu_{GSM}+\mu_{h,GSM})$. For GPRS sessions leaving the cell we have to distinguish if the packet arrival process of the terminated session is in bursty or non-bursty mode. In state (k, n, m ,r) are r GPRS sessions (out of m) in non-bursty mode and the remaining m-r sessions are in bursty mode. Therefore, the probability that the leaving session is in non-bursty mode is $\frac{r}{m}$ and that it is in bursty mode is $\frac{m-r}{m}$.

The arrival rate of IP packets is determined by the number of active GPRS sessions in the cell and by the state of the aggregated controlling (m+1)-state Markov chain. The average arrival rate of IP packets is given by the weighted sum of the arrival rates of r sessions in non-bursty mode and m-r sessions in bursty-mode. The same argument holds for the time spent in a particular state of the (m+1)-state Markov chain controlling the arrival process of IP packets. With rate $(m-r)\cdot\alpha$ the aggregated MMPP changes to a less bursty state and with rate $r\cdot\beta$ it changes to a more bursty state, respectively. In the service process for IP packets, the PDCHs are fairly shared by all packets in transfer up to a maximum of 8 PDCHs per IP packet ("multislot mode") and a maximum of 8 packets per PDCH [5]. With k packets residing in the BSC buffer a maximum of 8k PDCHs could be used for data transfer. We assume that at each time all free on-demand channels are allocated as PDCHs. Furthermore, the N_{GPRS} PDCHs are utilized for data transfer. This results in an overall number of N–n physical channel that are available for the transfer of GPRS packet data. Putting it altogether, we get a utilization of min(N − n, 8k) PDCHs in state (k, n, m, r).

The state space size of the Markov model (i.e., the dimension of matrix **Q**) is heavily influenced by the maximal number of concurrently active GPRS sessions M. Even if the state space is quite large (i.e., several hundreds of thousands states), using state-of-the-art numerical methods such as a sparse implementation of GMRES, the computation of the steady state solution $\boldsymbol{\pi}$ can be effectively computed on a modern PC with reasonable computational effort.

3.3. Derivation of Performance Measures

Recall that the arrival and service behavior for GSM calls and GPRS sessions constitute M/M/c/c queueing systems. Since the steady state solution for such a queue is known in closed-form, we can immediately derive performance measures. With

$$\rho_{GSM} = \frac{\lambda_{GSM} + \lambda_{h,GSM}}{\mu_{GSM} + \mu_{h,GSM}} \quad \text{and} \quad \rho_{GPRS} = \frac{\lambda_{GPRS} + \lambda_{h,GPRS}}{\mu_{GPRS} + \mu_{h,GPRS}}$$

the steady state solutions $\pi_{GSM,n}$ for n active GSM calls in the cell and $\pi_{GPRS,m}$ for m active GPRS sessions in the cell is given by:

$$\pi_{GSM,0} = \left(\sum_{n=0}^{N_{GSM}} \frac{\rho_{GSM}^n}{n!} \right)^{-1}, \ \pi_{GSM,n} = \pi_{GSM,0} \cdot \frac{\rho_{GSM}^n}{n!}, \quad \text{for } n = 1,2,...,N_{GSM} \qquad (1)$$

$$\pi_{GPRS,0} = \left(\sum_{m=0}^{M} \frac{\rho_{GPRS}^m}{m!} \right)^{-1}, \ \pi_{GPRS,m} = \pi_{GPRS,0} \cdot \frac{\rho_{GPRS}^m}{m!}, \quad \text{for } m = 1,2,...,M \qquad (2)$$

Using (1) and (2), we can calculate performance measures plotted in the figures presented in Section 4. Furthermore, we can use the steady-state solution to iteratively balance the handover flows of GSM calls and GPRS sessions in advance. From the solutions (1) and (2) we can calculate performance measures such as *Carried Voice Traffic (CVT)* and *Average Number of GPRS Users (AGU)*:

$$\text{CVT} = \sum_{n=1}^{N_{GSM}} n \cdot \pi_{GSM,n} \quad \text{and} \quad \text{AGU} = \sum_{m=1}^{M} m \cdot \pi_{GPRS,m}$$

The *GSM voice call blocking probability* and *GPRS user blocking probability* is simply given by $\pi_{GSM,N_{GSM}}$ and $\pi_{GPRS,M}$ respectively. Furthermore we can derive the *Average Packet Arrival Rate* by the product of the average number of GPRS users and the average packet arrival rate per session λ_{avg}.

Further performance measures can be obtained from the steady state solution $\boldsymbol{\pi}$ of the Markov model that can easily be computed numerically. The *Carried Data Traffic (CDT)* is the average number of channels in use for data transfer, i.e., PDCHs, and is given by:

$$\text{CDT} = \sum_{i=0}^{S_{max}} n(i) \cdot \pi_i$$

where n(i) is the number of PDCH utilized in state i and π_i is the steady-state probability of state i. The *Packet Loss Probability (PLP)* is the probability that an arriving IP packet finds a BSC buffer with already K packets queued and, thus, cannot be stored. It can be computed from the average packet arrival rate and the overall throughput of data packets:

$$\text{PLP} = 1 - \frac{\text{CDT} \cdot \mu_{service}}{\text{AGU} \cdot \lambda_{avg}}$$

A last performance measure of interest is the *Average Throughput per User (ATU)* that can be derived by the overall throughput of data packets and the average number of GPRS users in the cell:

$$\text{ATU} = \frac{\text{CDT} \cdot \mu_{service}}{\text{AGU}}$$

4. The Performance Study

Table 2 summarizes the base parameter settings underlying the performance experiments. We group the parameters into three classes: network model, mobility model, and traffic model. As base value, we assume that 5% of the arriving calls correspond to GPRS users and the remaining 95% are GSM calls. GSM call duration is set to 120 seconds and call dwell time to 60 seconds, so that users make 1-2 handovers on average. For GPRS sessions the average session duration is set to 5 minutes and the dwell time is 120 seconds. Thus, we assume longer "online times" and slower movement of GPRS users than for GSM users.

Table 2. Base parameter setting of the Markov model

Model Typ		Parameter	Base Value
Network Model		Number of physical channels, N	20
		Number of fixed PDCHs, N_{GPRS}	1
		Maximum number of GPRS users, M	20
		BSC buffer size, K	100 IP-packets
		Transfer rate for one PDCH (CS-2), $\mu_{service}$	13.4 Kbit/sec
Mobility Model		GSM handover arrival rate, $\lambda_{h,GSM}$	0.3/sec
		GPRS handover arrival rate, $\lambda_{h,GPRS}$	0.075/sec
		Average GSM voice call duration, $1/\mu_{GSM}$	120 sec
		Average GSM voice call dwell time, $1/\mu_{h,GSM}$	60 sec
		Average GPRS session duration, $1/\mu_{GPRS}$	300 sec
		Average GPRS session dwell time, $1/\mu_{h,GPRS}$	120 sec
Traffic Model	Users	GSM/GPRS call arrival rate, $\lambda = \lambda_{GSM} + \lambda_{GPRS}$	0.6/sec
		Percentage of GSM users	95%
		Percentage of GPRS users	5%
	Packet Data	Average arrival rate of data, λ_{avg}	6 Kbit/sec
		Degree of burstiness, B	5
		Average duration of bursty phase, $1/\alpha$	2 sec
		Average duration of non-bursty phase, $1/\beta$	20 sec

Recall that the major simplification of the Markov model presented in Section 3 stems from the choice of studying just one cell in isolation, instead of considering the entire cell cluster and the interactions among adjacent cells. This simplification relies on the assumption that under operating conditions of the cellular network (i.e., in steady-state) the average incoming handover flow is equal to the average outgoing handover flow. To validate this simplification of the Markov model, we additionally implemented a detailed simulator using the simulation library CSIM [10]. This simulator represents a cellular network comprising seven hexagonal cells and takes explicitly into account the handover procedures for GSM and GPRS users. Moreover, the architecture of the BTS and BSC is modeled in more detail than in the Markov model. The simulation results of the mid cell of the cell cluster are compared with corresponding results obtained from the Markov model. Confidence intervals with confidence level of 95% for simulation results are computed using batch means method.

Figure 2 plots curves for carried data traffic and packet loss probabilities for different percentages of GPRS users in comparison to the numerical results. The borders of the confidence intervals are drawn as dashed lines. Numerical results are drawn in solid lines. These curves clearly indicate that the simplifications introduced in the Markov model do not alter significantly the performance measures of interest. Thus, the Markov model is highly accurate and can effectively be utilized for studying the performance of GPRS.

The shape of the curves of Figure 2 can be explained as follows: For low traffic the fraction of the channel utilization corresponding to GPRS users increases up to 2.8 in case of 10% GPRS users. However, with increasing traffic the fraction of the channel utilization of GPRS users decreases because more and more GSM users occupy the radio resources. This is due to the assumption that GSM users have priority over GPRS users. Therefore,

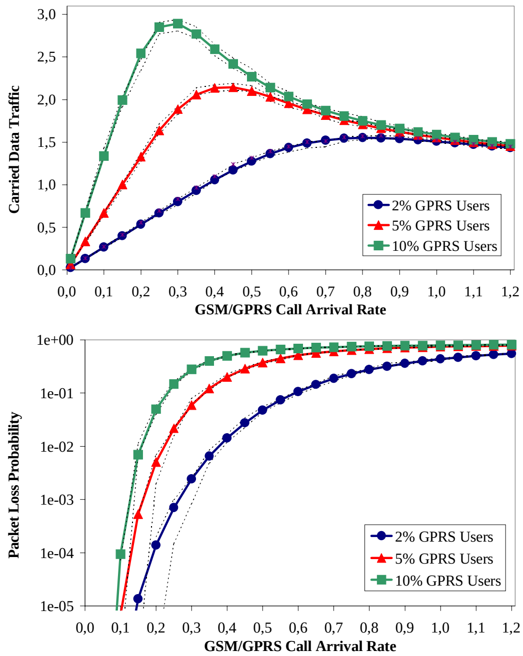

Figure 2. Validation of numerical results

for very high traffic the fraction of the channel utilization corresponding to GPRS users decreases to its minimum which corresponds to the one reserved PDCH.

Next we presents performance curves of the cellular mobile communication network derived from steady-state solutions of the Markov model. In particular, we investigate the impact of the number of PDCH reserved for GPRS users on the performance of the cellular network. This results give valuable hints for network designers on how many PDCH should be allocated for GPRS for a given amount of traffic in order to guarantee appropriate quality of service. In all curves the arrival rate of GSM and GPRS users is varied to study the cell under increasing traffic intensity.

Figure 3 presents a comparison of the mobile network for different system configurations. The comparison is made in two dimensions: the amount of GPRS users and the number of reserved PDCHs. In each curve we vary the number of reserved PDCHs (0, 1, 2, and 4) and the fraction of GPRS users among newly arriving calls (2%, 5%, and 10%). As performance measure, we consider the Average Throughput per User as defined in Section 3. From the curves we observe that under very low traffic intensity for GSM and GPRS users, each GPRS user reaches the maximum throughput of 6 Kbit/sec. With increasing load, the throughput per user decreases only slightly in case of 4 reserved PDCHs. This is in opposed to the case of no reserved PDCHs where the throughput approaches nearly zero. Comparing the different GPRS user populations, we discuss an example of high interest for network designers: Consider GPRS users with a QoS profile that allows a throughput degradation of at most

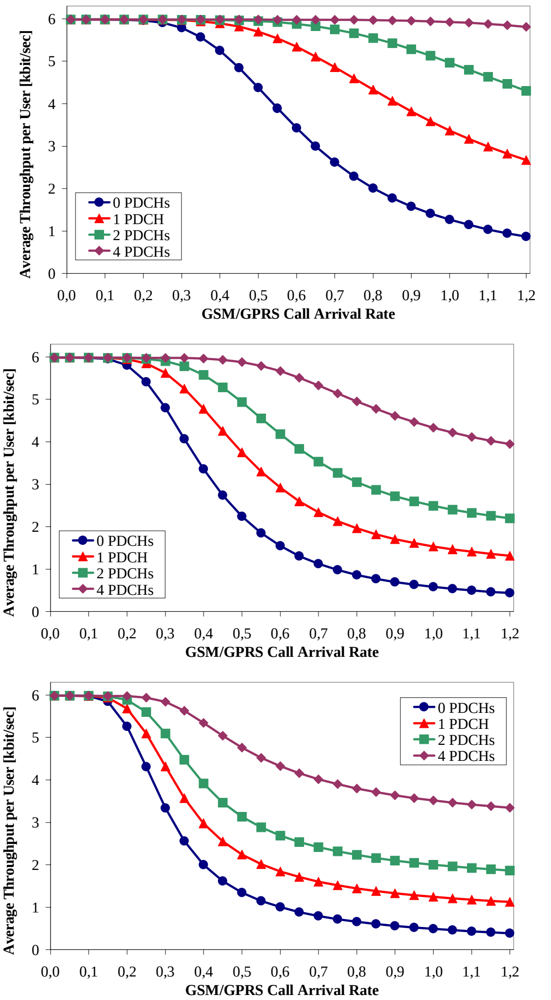

Figure 3. Throughput per User for 2%, 5% and 10% GPRS Users and different numbers of PDCHs

50%. Then, we can conclude that for 2% GPRS users the reservation of 2 PDCHs is sufficient even under heavy traffic conditions. However, for the case of 5% and 10% GPRS users, 4 PDCHs are needed to meet the quality of service requirements. Reserving no PDCH for GPRS results in unacceptable degradation of quality of service.

In an additional experiment, we study the performance loss in the GSM voice service due to the introduction of GPRS. Figure 4 plots the carried voice traffic and voice blocking probability for different numbers of reserved PDCHs. The presented curves indicate that the decrease in channel capacity and, thus, the increase of the blocking probability of the GSM voice service is negligible compared to the benefit of reserving additional PDCHs for GPRS users.

Figure 5 presents curves for average number of GPRS users in the cell and blocking probabilities of GPRS session requests due to reaching the limit of M active GPRS sessions. We observe that for 2% GPRS users the

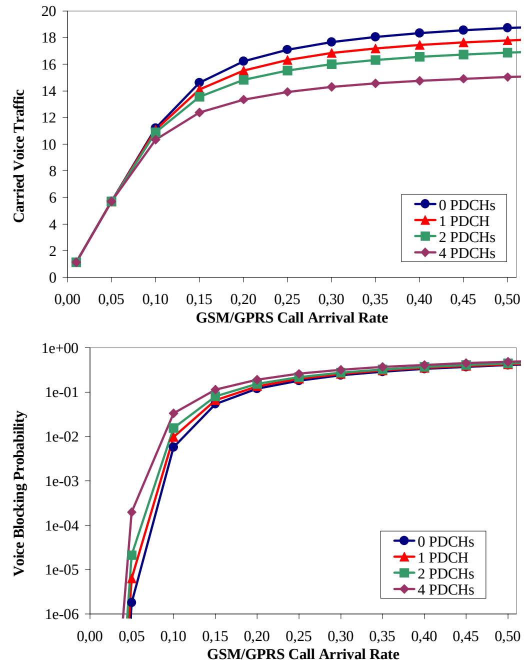

Figure 4. Influence of GPRS on GSM voice service

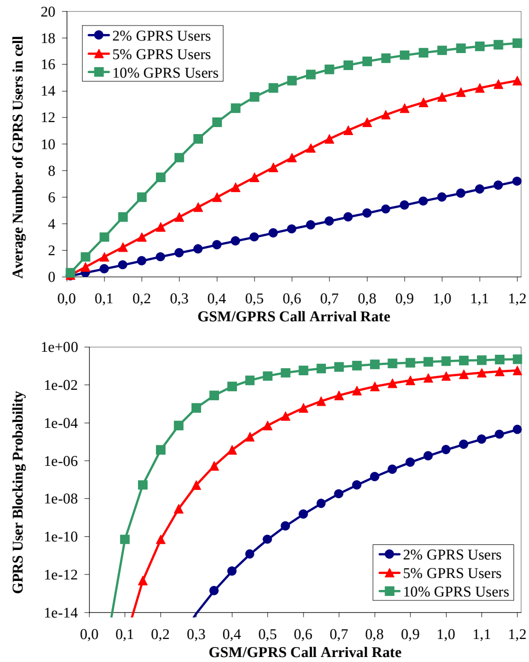

Figure 5. Average number of GPRS users in cell and GPRS user blocking probability

maximum number of 20 active GPRS sessions is not reached. Therefore, the blocking probability remains very low. For 10% GPRS users and increasing call arrival rate, the average number of sessions approaches its maximum. Thus, some GPRS users will be rejected. It is important to note that the curves of Figure 5 can be utilized for determining the average number of GPRS users in the cell for a given call arrival rate. In fact, together with the curves of Figure 3, we can provide estimates for the maximum number of GPRS users that can be managed by the cell without degradation of quality of service. For example, for 5% GPRS users and 4 PDCHs reserved, the maximum throughput of 6 Kbit/sec is achieved until the call arrival rate exceeds 0.4 calls per second, i.e., until there are on the average 6 active GPRS users in the cell.

Figure 6 investigates the impact of the maximum number of GPRS user per cell to the performance of GPRS. Of course, the expected number of GPRS users should be less than the maximum number in order to avoid the rejection of new GPRS sessions. On the other hand, the maximum number of active GPRS sessions must be limited for guaranteeing quality of service for every active GPRS session even under high traffic. The tradeoff between increasing performance for allowing more active GPRS sessions and the increasing blocking probability for GPRS users is illustrated by the curves of Figure 6.

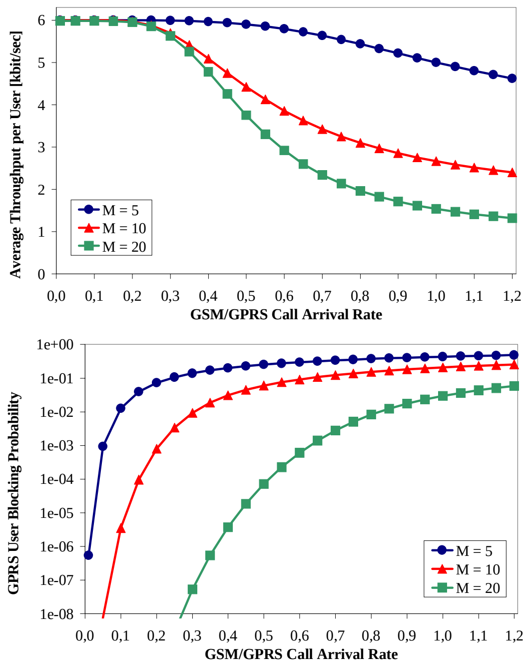

Figure 6. Average throughput per user and GPRS user blocking probability for different GPRS user limits

5. Conclusions

This paper presented a comprehensive performance study of the radio resource sharing by circuit switched GSM connections and packet switched GPRS sessions under a dynamic channel allocation scheme. We assumed a reserved number of physical channels permanently allocated to GPRS and the remaining channels to be on-demand channels that can be used by GSM voice service and GPRS packets. We investigate the impact of the number of packet data channels reserved for GPRS users on the performance of the cellular network.

Our results give valuable hints for network designers on how many packet data channels should be allocated for GPRS for a given amount of traffic in order to guarantee appropriate quality of service. The performance results are derived from the steady-state analysis of a Markov model. A validation of the Markov model with a detailed simulator shows that almost all performance curves for measures of interest derived from the Markov model lie in the confidence intervals of the corresponding curve of the simulator. The presented Markov model can be analyzed with few minutes of CPU solution time on a modern PC with high numerical accuracy whereas the simulator requires simulation runs in the order of hours.

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